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# Event-Based Recursive Filter for Time-Varying Systems with Randomly Occurring Nonlinearities and One-Step Time Delay

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## Abstract

**In this paper, we study a class of filtering problems for time-varying systems with stochastic non-linearities and one-step delays. Randomly occurring nonlinearities are described by introducing a random sequence that obeys the Bernoulli distribution. For the one-step time lag phenomenon, it is characterized by random variables obeying the Bernoulli distribution. In order to deal with the one-step time lag, this paper will augment the proposed system equation. In order to reduce the network transmission pressure, the event trigger transmission mechanism is introduced. The main purpose of this paper is to propose a recursive filtering algorithm with one-step time lag. Since the estimated error covariance matrix cannot be accurately calculated, the upper bound of the error covariance matrix is found by scaling, and the upper bound trace is minimized by designing the filter gain matrix. Finally, an example simulation experiment is given to illustrate the effectiveness of the algorithm.**

## Keywords

**Time-Varying System, One-Step Time Delay, Randomly Occurring Nonlinearities, Filter**

# 具有随机发生非线性和一步时滞的时变系统的滤波问题

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## 摘　要

**本文研究了一类具有随机发生非线性和一步时滞的时变系统的滤波问题。通过引入一个服从伯努利分布的随机序列，来描述随机发生的非线性。对于一步时滞现象，则是利用服从伯努利分布的随机变量进行刻画，为了处理一步时滞，本文将对所提出的系统方程进行增广。为了减少网络传输压力，文中引入了事件触发传输机制。本文的主要目的是提出一种具有一步时滞的递推滤波算法。由于估计误差协方差矩阵不能准确计算得出，于是通过放缩找到误差协方差矩阵的上界，并且通过设计滤波增益矩阵使得该上界的迹达到最小。最后，给出一个算例仿真实验来说明该算法的有效性。**



## 关键词

**时变系统，一步时滞，随机发生非线性，滤波**




## 1. 引言

作为现代控制理论的一个重要分支，Kalman 滤波得到了国内外专家学者的广泛研究。卡尔曼滤波是一种最优算法，可以应用于受随机噪声干扰的网络化传输系统中。准确的说，卡尔曼滤波是一种算法，且该算法具有递推形式，可用于多变量非平稳的随机过程。该算法是在 20 世纪 60 年代由著名科学家 R. E. Kalman 提出的，卡尔曼滤波的提出，在当时的学术界引起了强烈的反响，使得控制理论得到了质的提高。卡尔曼滤波算法的优点在于，其本身是一种递推估计算法，不需要存储所有的观测信息，只需上一个估计时刻以及当前时刻的信息即可求出当前时刻的估计值，而且计算相对方便，存储量也相对较小，在实际工程中便于实现。不得不说的是，卡尔曼滤波算法针对的系统往往都是线性系统，而在实际工程中，系统本身往往都是非线性的，而且非线性大都是随机发生的。以此，带有随机发生非线性系统的卡尔曼滤波问题引起了广泛的关注[1]-[23]。

经过几十年的研究发展，理论上对只含有系统状态方程和测量输出方程的卡尔曼滤波问题的研究已经比较成熟。但是在实际工程问题中，当数据在网络化传输过程中，往往会发生一定的网络诱导现象，比如时滞、测量丢失、数据包丢失、数据包传输错序……网络诱导现象的产生，又重新引起了学者们对卡尔曼滤波的研究兴趣。本文中所考虑的网络化诱导现象为一步时滞，在已有的研究成果中，[9]讨论了具有时滞的卡尔曼滤波问题，但是值得一体的是，上述文献中所讨论的是线性系统，对于非线性系统的一步时滞问题的研究目前尚处于萌芽状态。基于此本文主要考虑具有一步时滞和随机发生非线性系统的卡尔曼滤波问题。

当数据通过媒介进行交换和传输时，由于网络带宽资源受限，往往会发生各类的网络诱导现象。为了减少网络传输压力，节省网络带宽资源，在实际网络化传输系统中，往往会加入相应的传输协议，如 Round-Robin 协议，Try Once Discard 协议等等，本文中所考虑的是事件触发传输协议，在以往的研究中，研究者们考虑的往往都是时间触发协议，但是时间触发协议不能充分的利用有限的网络资源，而且再进行数据传输时，数据中包含的新信息特别有限，对原本就有限的网络资源造成了极大地浪费。不同于时

间触发，事件触发的基本思想是“按需分配”，只有在系统满足一定的条件时，数据才会进行传递。引入事件触发协议，可以有效地利用通信资源，因此，事件触发协议引起国内外学者的广泛关注[24] [25] [26] [27] [28]。

## 2. 模型建立

在本文中，建立如下的具有随机发生非线性和一步时滞的时变系统动态模型：

$$\begin{aligned} &\vec{x}_{k+1} = \vec{A}_k\vec{x}_k + \alpha_k\vec{f}\left(\vec{x}_k\right) + \vec{B}_k\vec{\omega}_k \\ &\vec{y}_k = \vec{C}_k\vec{x}_k + \vec{v}_k \\ &y_k = \lambda_k\vec{y}_k + \left(1-\lambda_k\right)\vec{y}_{k-1} \end{aligned} \tag{1}$$

其中，$\vec{x}_k$ 代表代表 $k$ 时刻系统的状态向量，$\vec{y}_k$ 代表系统的测量输出，$\vec{f}\left(\vec{x}_k\right)$ 为非线性函数；$\vec{\omega}_k$ 是均值为零方差为 $\vec{Q}_k$ 的过程噪声；$\vec{v}_k$ 是均值为零方差为 $\vec{R}_k$ 的测量噪声；$\vec{A}_k$、$\vec{B}_k$、$\vec{C}_k$ 均为已知的系统矩阵，$\alpha_k$ 和 $\lambda_k$ 均为服从伯努利分布的随机变量，分别刻画随机发生的非线性与随机发生的一步时滞，并满足以下条件：

$$\begin{aligned} &\text{Prob}\left\{\alpha_k = 1\right\} = \mathbb{E}\left\{\alpha_k\right\} = \bar{\alpha}_k \\ &\text{Prob}\left\{\alpha_k = 0\right\} = 1-\bar{\alpha}_k \\ &\text{Prob}\left\{\lambda_k = 1\right\} = \mathbb{E}\left\{\lambda_k\right\} = \bar{\lambda}_k \\ &\text{Prob}\left\{\lambda_k = 0\right\} = 1-\bar{\lambda}_k \end{aligned} \tag{2}$$

其中，$\bar{\alpha}_k$ 和 $\bar{\lambda}_k$ 分别代表已知的发生概率。

假设：非线性函数 $\vec{f}\left(\vec{x}_k\right)$ 满足如下的利普希茨条件：

$$\left\|\vec{f}\left(\vec{x}_k\right) - \vec{f}\left(\vec{z}_k\right)\right\| \le l\left\|\vec{x}_k - \vec{z}_k\right\| \tag{3}$$

为了计算简便，进入如下形式的增广：

$$\begin{aligned} &x_k = \begin{bmatrix}\vec{x}_k \\ \vec{x}_{k-1}\end{bmatrix}, f\left(x_k\right) = \begin{bmatrix}\vec{f}\left(\vec{x}_k\right) \\ 0\end{bmatrix}, \bar{I} = \begin{bmatrix}0 & 0 \\ I & 0\end{bmatrix}, A_k = \begin{bmatrix}\vec{A}_k & 0 \\ 0 & 0\end{bmatrix}, \\ &B_k = \begin{bmatrix}\vec{B}_k \\ 0\end{bmatrix}, C_k = \begin{bmatrix}\vec{C}_k & 0 \\ 0 & \vec{C}_{k-1}\end{bmatrix}, v_k = \begin{bmatrix}\vec{v}_k \\ \vec{v}_{k-1}\end{bmatrix}, \\ &\Upsilon_k = \begin{bmatrix}\lambda_k I & \left(1-\lambda_k\right)I\end{bmatrix}, \vec{\omega}_k = \omega_k \end{aligned}$$

得到增广后的时变系统动态模型：

$$\begin{aligned} &x_{k+1} = \vec{A}_k x_k + \alpha_k f\left(x_k\right) + B_k\omega_k \\ &\bar{y}_k = \Upsilon_k\left(C_k x_k + v_k\right) \end{aligned} \tag{4}$$

其中 $\bar{A}_k = \left(A_k + \bar{I}\right)$，增广后的测量噪声 $v_k$ 具有如下的统计特性：

$$\begin{aligned} &E\left\{v_k\right\} = 0 \\ &E\left\{v_k v_l^{\mathrm{T}}\right\} = R_k\delta_{k-l} + R_{k,k+1}\delta_{k-l-1} + R_{k,k+1}\delta_{k-l+1} \end{aligned}$$

其中

$$R_k = \begin{bmatrix}\vec{R}_k & 0 \\ 0 & \vec{R}_{k-1}\end{bmatrix}, R_{k,k-1} = \begin{bmatrix}0 & 0 \\ \vec{R}_{k-1} & 0\end{bmatrix}, R_{k,k+1} = \begin{bmatrix}0 & \vec{R}_k \\ 0 & 0\end{bmatrix}$$

令 $Q_k = \vec{Q}_k$。

在网络传输过程中，为了减少网络传输压力，节省网络带宽资源，通常会引入相应的通信协议。在本文中，引入如下形式的事件触发传输机制：

$$\left(y_{k+l} + y_{k_t}\right)^{\mathrm{T}}\left(y_{k+l} - y_{k_t}\right) > \theta \tag{5}$$

式中 $y_{k_t}$ 是最近事件触发时刻的测量输出，$\theta$ 是已知的调节阈值，那么时变动态系统在 $k$ 时刻的实际输出如下所示：

$$\tilde{y}_k = y_{k_t}, k \in \left\{k_i, k_i + 1, \cdots, k_{i+1} - 1\right\}$$

其中 $\tilde{y}_k$ 为 $k$ 时刻的实际输出值。

针对上述增广系统，构造如下形式的滤波器：

$$\begin{aligned}\hat{x}_{k+1|k} &= \bar{A}_k \hat{x}_{k|k} + \bar{\alpha}_k f\left(\hat{x}_{k|k}\right) \\ \hat{x}_{k+1|k+1} &= \hat{x}_{k+1|k} + K_{k+1}\left(\tilde{y}_{k+1} - \bar{\Upsilon}_{k+1} C_{k+1} \hat{x}_{k+1|k}\right)\end{aligned} \tag{6}$$

式中 $\hat{x}_{k|k}$ 是 $x_k$ 在 $k$ 时刻的状态估计，$\hat{x}_{k+1|k}$ 是 $x_k$ 在 $k$ 时刻的一步预测，$\hat{x}_{k+1|k+1}$ 是 $k+1$ 时刻的状态估计，$K_{k+1}$ 是 $K_{k+1}$ 时刻的滤波增益矩阵，$\left[\bar{\lambda}_{k+1} I \quad \left(1-\bar{\lambda}_{k+1}\right) I\right] = \bar{\Upsilon}_{k+1}$。

本文主要有以下两个目的。第一，针对具有随机发生非线性的时变系统设计形如(6)的滤波器，得到滤波误差协方差矩阵的上界，即找到正定矩阵 $\Sigma_{k+1|k+1}$ 满足如下关系式：

$$\mathbb{E}\left\{\left(x_{k+1} - \hat{x}_{k+1|k+1}\right)\left(x_{k+1} - \hat{x}_{k+1|k+1}\right)^{\mathrm{T}}\right\} \le \Sigma_{k+1|k+1} \tag{7}$$

第二，通过设计适当的滤波器增益使得滤波误差协方差矩阵上界的迹达到最小。

## 3. 主要结论

首先，介绍如下引理：

引理 1：对于两个实列向量 $a, b \in R^n$，则下面的不等式成立：

$$ab^{\mathrm{T}} + ba^{\mathrm{T}} \le \varepsilon aa^{\mathrm{T}} + \varepsilon^{-1} bb^{\mathrm{T}}$$

其中 $\varepsilon$ 是已知的正数。

由公式(4)减去公式(6)，可得一步预测误差表达式如下：

$$\tilde{x}_{k+1|k} = \bar{A}\tilde{x}_{k|k} + \bar{\alpha}_k\left[f\left(x_k\right) - f\left(\hat{x}_{k|k}\right)\right] + \tilde{\alpha}_k f\left(x_k\right) + B_k \omega_k \tag{8}$$

式中 $\tilde{\alpha}_k = \alpha_k - \bar{\alpha}_k$。

同样地，得到滤波误差表达式：

$$\begin{aligned}\tilde{x}_{k+1|k+1} = &\left(I - K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)\bar{x}_{k+1/k} + K_{k+1}\left(\tilde{y}_{k+1} - y_{k+1}\right) \\ &+ K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}x_{k+1} - K_{k+1}\Upsilon_{k+1}\nu_{k+1}\end{aligned} \tag{9}$$

其中 $\tilde{\Upsilon}_{k+1} = \Upsilon_{k+1} - \bar{\Upsilon}_{k+1}$。

引理 2：增广系统的状态协方差矩阵上界 $X_{k+1} = \mathbb{E}\left\{x_{k+1}x_{k+1}^{\mathrm{T}}\right\}$ 的表达式如下：

$$X_{k+1} \le \left(1+\varepsilon\right)\bar{A}_k X_k \bar{A}_k^{\mathrm{T}} + \left(1+\varepsilon^{-1}\right) l^2 \operatorname{tr}\left(\bar{X}_k\right) I + B_k Q_k B_k^{\mathrm{T}} := \bar{X}_{k+1} \tag{10}$$

上式中 $\varepsilon$ 是已知的正数。

证明：增广系统的状态协方差矩阵 $X_{k+1}=\mathbb{E}\left\{x_{k+1}x_{k+1}^{\mathrm{T}}\right\}$ 的递推表达式如下：

$$X_{k+1}=\bar{A}_k X_k \bar{A}_k+\bar{\alpha}_k\mathbb{E}\left\{f\left(x_k\right)f^{\mathrm{T}}\left(x_k\right)\right\}+B_k Q_k B_k^{\mathrm{T}}+\Lambda_1+\Lambda_1^{\mathrm{T}} \tag{11}$$

其中 $\Lambda_1=\mathbb{E}\left\{\alpha_k f\left(x_k\right)x_k^{\mathrm{T}}\bar{A}_k^{\mathrm{T}}\right\}$

应用引理 1 可得

$$\Lambda_1+\Lambda_1^{\mathrm{T}}\le\varepsilon\bar{A}_k X_k\bar{A}_k^{\mathrm{T}}+\varepsilon^{-1}\bar{\alpha}_k\mathbb{E}\left\{f\left(x_k\right)f^{\mathrm{T}}\left(x_k\right)\right\} \tag{12}$$

将上式带入到公式(11)中

$$X_{k+1}\le\left(1+\varepsilon\right)\bar{A}_k X_k\bar{A}_k^{\mathrm{T}}+\left(1+\varepsilon^{-1}\right)\bar{\partial}_k\mathbb{E}\left\{f\left(x_k\right)f^{\mathrm{T}}\left(x_k\right)\right\}+B_k Q_k B_k^{\mathrm{T}} \tag{13}$$

根据不等式性质得到

$$f\left(x_k\right)f^{\mathrm{T}}\left(x_k\right)\le\left\|f\left(x_k\right)\right\|^2 I=f\left(x_k\right)f^{\mathrm{T}}\left(x_k\right)I \tag{14}$$

$$E\left\{f\left(x_k\right)f^{\mathrm{T}}\left(x_k\right)\right\}\le l^2\mathbb{E}\left\{x_k x_k^{\mathrm{T}}\right\}=l^2\operatorname{tr}\left(X_k\right) \tag{15}$$

将上式代入公式(13)，得到增广系统的状态协方差上界表达式

$$X_{k+1}\le\left(1+\varepsilon\right)\bar{A}_k X_k\bar{A}_k^{\mathrm{T}}+\left(1+\varepsilon^{-1}\right)l^2\operatorname{tr}\left(X_k\right)I+B_k Q_k B_k^{\mathrm{T}} \tag{16}$$

定理 1：一步预测误差协方差矩阵 $P_{k+1|k}=\mathbb{E}\left\{\bar{x}_{k+1|k}\bar{x}_{k+1|k}^{\mathrm{T}}\right\}$ 的递推表达式如下所示

$$\begin{aligned}P_{k+1|k}=&\bar{A}_k P_{k|k}\bar{A}_k^{\mathrm{T}}+\bar{\alpha}_k^2\mathbb{E}\left\{\left[f\left(x_k\right)-f\left(\hat{x}_{k|k}\right)\right]\left[f\left(x_k\right)-f\left(\hat{x}_{k|k}\right)^{\mathrm{T}}\right]\right\}\\&+\bar{\alpha}_k\left(1-\bar{\alpha}_k\right)\mathbb{E}\left\{f\left(x_k\right)f^{\mathrm{T}}\left(x_k\right)\right\}+B_k Q_k B_k^{\mathrm{T}}+\Lambda_2\end{aligned} \tag{17}$$

其中：$\Lambda_2=\mathbb{E}\left\{\bar{\alpha}_k\left[f\left(x_k\right)-f\left(\hat{x}_{k/k}\right)\right]\tilde{x}_{k|k}^{\mathrm{T}}\bar{A}_k^{\mathrm{T}}\right\}$

$P_{k|k}=\mathbb{E}\left\{\tilde{x}_{k|k}\tilde{x}_{k|k}^{\mathrm{T}}\right\}$ 为滤波误差协方差。

证明：根据一步预测误差表达式(8)，由于 $\mathbb{E}\left\{\tilde{\alpha}_k\right\}=0$，$\mathbb{E}\left\{\omega_k\right\}=0$，很容易可得定理 1，因此定理 1 证明过程从略。

定理 2：滤波误差协方差 $P_{k+1|k+1}=\mathbb{E}\left\{\tilde{x}_{k+1|k+1}\tilde{x}_{k+1|k+1}^{\mathrm{T}}\right\}$ 的递推表达式如下：

$$\begin{aligned}P_{k+1|k+1}=&\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)P_{k+1|k}\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)^{\mathrm{T}}\\&+K_{k+1}\mathbb{E}\left\{\left(\tilde{y}_{k+1}-y_{k+1}\right)\left(\tilde{y}_{k+1}-y_{k+1}\right)^{\mathrm{T}}\right\}K_{k+1}^{\mathrm{T}}\\&+\mathbb{E}\left\{\left(K_{k+1}\tilde{\Upsilon}_{k+1}C_{k+1}x_{k+1}\right)\left(K_{k+1}\tilde{\Upsilon}_{k+1}C_{k+1}x_{k+1}\right)^{\mathrm{T}}\right\}\\&+K_{k+1}\mathbb{E}\left\{\Upsilon_{k+1}v_{k+1}v_{k+1}^{\mathrm{T}}\Upsilon_{k+1}\right\}K_{k+1}^{\mathrm{T}}+\Lambda_3+\Lambda_4\end{aligned} \tag{18}$$

其中 $\Lambda_3=\mathbb{E}\left\{\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)\tilde{x}_{k+1|k}\left(\tilde{y}_{k+1}-y_{k+1}\right)^{\mathrm{T}}K_{k+1}^{\mathrm{T}}\right\}$，

$$\Lambda_4=\mathbb{E}\left\{K_{k+1}\left(\bar{y}_{k+1}-y_{k+1}\right)v_{k+1}^{\mathrm{T}}\Upsilon_{k+1}^{\mathrm{T}}K_{k+1}^{\mathrm{T}}\right\}$$

证明：考虑到 $\mathbb{E}\left\{\tilde{\Upsilon}_k\right\}=0$ 和 $\mathbb{E}\left\{v_k\right\}=0$，并且根据滤波误差表达式(9)，定理 2 可以很容易得出，故而证明从略。

定理 3：根据定理 1 和定理 2 给出的一步预测误差协方差矩阵表达式和滤波误差协方差矩阵表达式。取 $\eta$，$\mu_1$，$\mu_2$ 分别为大于零的数，如果如下的类黎卡提差分方程：

$$\Sigma_{k+1|k}=(1+\eta)\bar{A}_k\Sigma_{k|k}\bar{A}_k^{\mathrm{T}}+\left(1+\eta^{-1}\right)\bar{\alpha}_k^2 l^2\operatorname{tr}\left(\Sigma_{k|k}\right)I \\ +\bar{\alpha}_k\left(1-\bar{\alpha}_k\right)l^2\operatorname{tr}\left(\bar{X}_k\right)I+B_kQ_kB_k^{\mathrm{T}} \tag{19}$$

$$\Sigma_{k+1|k+1}=(1+\mu_1)\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)\Sigma_{k+1|k}\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)^{\mathrm{T}} \\ +\left(1+\mu_1^{-1}+\mu_2\right)\theta K_{k+1}K_{k+1}^{\mathrm{T}}+K_{k+1}\bar{\lambda}_{k+1}\left(1-\bar{\lambda}_{k+1}\right)l^2\operatorname{tr}\left(\bar{X}_{k+1}\right) \\ \times K_{k+1}\bar{H}_{k+1}C_{k+1}C_{k+1}^{\mathrm{T}}\bar{H}_{k+1}^{\mathrm{T}}K_{k+1}^{\mathrm{T}}+\left(1+\mu_2^{-1}\right)\bar{\lambda}_{k+1}K_{k+1}\bar{H}_{k+1}R_{k+1}\bar{H}_{k+1}^{\mathrm{T}}K_{k+1}^{\mathrm{T}} \tag{20}$$

在初始条件 $\Sigma_{0|0}=P_{0|0}>0$ 下有正解 $\Sigma_{k+1|k}$ 和 $\Sigma_{k+1|k+1}$，则矩阵 $\Sigma_{k+1|k+1}$ 是 $P_{k+1|k+1}$ 的上界。

证明：对定理 1 中的交叉项 $\Lambda_2$ 应用引理 1 可得

$$\Lambda_2+\Lambda_2^{\mathrm{T}}=\eta\bar{A}_kP_{k|k}\bar{A}_k^{\mathrm{T}}+\eta^{-1}\bar{\alpha}_k^2\mathbb{E}\left\{\left[f(x_k)-f\left(\hat{x}_{k|k}\right)\right]\left[f(x_k)-f\left(\hat{x}_{k|k}\right)^{\mathrm{T}}\right]\right\} \tag{21}$$

则公式(17)可化简为如下形式

$$p_{k+1|k}\le(1+\eta)\bar{A}_kP_{k|k}\bar{A}_k^{\mathrm{T}}+\left(1+\eta^{-1}\right)\bar{\alpha}_k^2 \\ \times\mathbb{E}\left\{\left[f(x_k)-f\left(\hat{x}_{k/k}\right)\right]\left[f(x_k)-f\left(\hat{x}_{k/k}\right)\right]^{\mathrm{T}}\right\} \\ +\bar{\alpha}_k\left(1-\bar{\alpha}_k\right)\mathbb{E}\left\{f(x_k)f^{\mathrm{T}}(x_k)\right\}+B_kQ_kB_k^{\mathrm{T}} \tag{22}$$

由于

$$\left[f(x_k)-f(\hat{x}_k)\right]\left[f(x_k)-f(\hat{x}_k)\right]^{\mathrm{T}}\le\left\|f(x_k)-f(\hat{x}_k)\right\|^2 I=\left[f(x_k)-f(\hat{x}_k)\right]^{\mathrm{T}}\left[f(x_k)-f(\hat{x}_k)\right]I \tag{23}$$

故而

$$\mathbb{E}\left\{\left[f(x_k)-f(\hat{x}_k)\right]^{\mathrm{T}}\left[f(x_k)-f(\hat{x}_k)\right]\right\}\le l^2\mathbb{E}\left\{\tilde{x}_k^{\mathrm{T}}\tilde{x}_k\right\}=l^2\operatorname{tr}\left(P_{k|k}\right) \tag{24}$$

将上式代入(22)得

$$P_{k+1|k}\le(1+\eta)\bar{A}_kP_{k|k}\bar{A}_k^{\mathrm{T}}+\left(1+\eta^{-1}\right)\bar{\alpha}_k^2l^2\operatorname{tr}\left(P_{k|k}\right)I \\ +\bar{\alpha}_k\left(1-\bar{\alpha}_k\right)l^2\operatorname{tr}\left(X_k\right)I+B_kQ_kB_k^{\mathrm{T}} \tag{25}$$

同样地，对于定理 2 中的交叉项 $\Lambda_3$ 和 $\Lambda_4$，应用引理 1 可得

$$\Lambda_3+\Lambda_3^{\mathrm{T}}\le\mu_1\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)P_{k+1|k}\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)^{\mathrm{T}} \\ +\mu_1^{-1}K_{k+1}\mathbb{E}\left\{\left(\tilde{y}_{k+1}-y_{k+1}\right)\left(\tilde{y}_{k+1}-y_{k+1}\right)^{\mathrm{T}}\right\}K_{k+1}^{\mathrm{T}} \tag{26}$$

$$\Lambda_4+\Lambda_4^{\mathrm{T}}\le\mu_2K_{k+1}\mathbb{E}\left\{\left(\tilde{y}_{k+1}-y_{k+1}\right)\left(\tilde{y}_{k+1}-y_{k+1}\right)^{\mathrm{T}}\right\}K_{k+1}^{\mathrm{T}} \\ +\mu_2^{-1}K_{k+1}\mathbb{E}\left\{\Upsilon_{k+1}v_{k+1}v_{k+1}^{\mathrm{T}}\Upsilon_{k+1}\right\}K_{k+1}^{\mathrm{T}} \tag{27}$$

将上述两式代入(18)中，

$$P_{k+1|k+1}\le(1+\mu_1)\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)P_{k+1|k}\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)^{\mathrm{T}} \\ +\left(1+\mu_1^{-1}+\mu_2\right)K_{k+1}\mathbb{E}\left\{\left(\tilde{y}_{k+1}-y_{k+1}\right)\left(\tilde{y}_{k+1}-y_{k+1}\right)^{\mathrm{T}}\right\}K_{k+1}^{\mathrm{T}} \\ +K_{k+1}\mathbb{E}\left\{\left(\tilde{\Upsilon}_{k+1}C_{k+1}x_{k+1}\right)\left(\tilde{\Upsilon}_{k+1}C_{k+1}x_{k+1}\right)^{\mathrm{T}}\right\}K_{k+1}^{\mathrm{T}} \\ +\left(1+\mu_2^{-1}\right)K_{k+1}\mathbb{E}\left\{\Upsilon_{k+1}v_{k+1}v_{k+1}^{\mathrm{T}}\Upsilon_{k+1}\right\}K_{k+1}^{\mathrm{T}} \tag{28}$$

考虑到事件触发表达式(5)，上式的第二项可进行如下处理

$$K_{k+1}\mathbb{E}\left\{\left(\tilde{y}_{k+1}-y_{k+1}\right)\left(\tilde{y}_{k+1}-y_{k+1}\right)^{\mathrm{T}}\right\}K_{k+1}^{\mathrm{T}}\le\theta K_{k+1}K_{k+1}^{\mathrm{T}} \tag{29}$$

将引理 2 应用到上述公式(28)的后两项：

$$\begin{aligned}&K_{k+1}\left\{\tilde{\Upsilon}_{k+1}C_{k+1}x_{k+1}x_{k+1}^{\mathrm{T}}C_{k+1}^{\mathrm{T}}\tilde{\Upsilon}_{k+1}^{\mathrm{T}}\right\}K_{k+1}^{\mathrm{T}}\\&\le\bar{\lambda}_{k+1}\left(1-\bar{\lambda}_{k+1}\right)l^2\operatorname{tr}\left(X_{k+1}\right)K_{k+1}\bar{H}_{k+1}C_{k+1}C_{k+1}^{\mathrm{T}}\bar{H}_{k+1}^{\mathrm{T}}K_{k+1}^{\mathrm{T}}\end{aligned} \tag{30}$$

$$K_{k+1}\mathbb{E}\left\{\Upsilon_{k+1}v_{k+1}v_{k+1}^{\mathrm{T}}\Upsilon_{k+1}\right\}K_{k+1}^{\mathrm{T}}\le\bar{\lambda}_{k+1}K_{k+1}\bar{H}_{k+1}R_{k+1}\bar{H}_{k+1}^{\mathrm{T}}K_{k+1}^{\mathrm{T}} \tag{31}$$

其中 $\bar{H}_{k+1}=\begin{bmatrix}I & -I\end{bmatrix}$。

将公式(29)~(31)代入(28)

$$\begin{aligned}P_{k+1|k+1}\le&\left(1+\mu_1\right)\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)P_{k+1|k}\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)^{\mathrm{T}}\\&+\left(1+\mu_1^{-1}+\mu_2\right)\theta K_{k+1}K_{k+1}^{\mathrm{T}}+K_{k+1}\bar{\lambda}_{k+1}\left(1-\bar{\lambda}_{k+1}\right)\\&\times l^2\operatorname{tr}\left(X_{k+1}\right)K_{k+1}\bar{H}_{k+1}C_{k+1}C_{k+1}^{\mathrm{T}}\bar{H}_{k+1}^{\mathrm{T}}K_{k+1}^{\mathrm{T}}\\&+\left(1+\mu_2^{-1}\right)\bar{\lambda}_{k+1}K_{k+1}\bar{H}_{k+1}R_{k+1}\bar{H}_{k+1}^{\mathrm{T}}K_{k+1}^{\mathrm{T}}\end{aligned} \tag{32}$$

定理 3 证毕。

定理 4：如果滤波估计增益按如下形式给出，则滤波误差协方差矩阵上界 $\Sigma_{k+1|k+1}$ 的迹可达到最小。

$$K_{k+1}=\left(1+\mu_1\right)\Sigma_{k+1/k}C_{k+1}\bar{\Upsilon}_{k+1}\left\{\Psi_{k+1}\right\}^{-1} \tag{33}$$

其中

$$\begin{aligned}\Psi_{k+1}=&\left(1+\mu_1\right)\bar{\Upsilon}_{k+1}C_{k+1}\Sigma_{k+1|k}C_{k+1}^{\mathrm{T}}\bar{\Upsilon}_{k+1}^{\mathrm{T}}+\left(1+\mu_1^{-1}+\mu_2\right)\theta I\\&+\bar{\lambda}_{k+1}\left(1-\bar{\lambda}_{k+1}\right)l^2\operatorname{tr}\left(X_{k+1}\right)\bar{H}_{k+1}C_{k+1}C_{k+1}^{\mathrm{T}}\bar{H}_{k+1}^{\mathrm{T}}\\&+\left(1+\mu_2^{-1}\right)\bar{\lambda}_{k+1}\bar{H}_{k+1}R_{k+1}\bar{H}_{k+1}^{\mathrm{T}}\end{aligned}$$

证明：

$$\frac{\partial\operatorname{tr}\left(\Sigma_{k+1|k+1}\right)}{\partial K_{k+1}}=-2\left(1+\mu_1\right)\left(I-K_{k+1}\bar{\Upsilon}_{k+1}C_{k+1}\right)\Sigma_{k+1|k}C_{k+1}^{\mathrm{T}}\bar{\Upsilon}_{k+1}^{\mathrm{T}}+2K_{k+1}\left\{\Psi_{k+1}\right\} \tag{34}$$

令 $\dfrac{\partial\Sigma_{k+1|k+1}}{\partial K_{k+1}}=0$，

可得 $K_{k+1}=\left(1+\mu_1\right)\Sigma_{k+1/k}C_{k+1}\bar{\Upsilon}_{k+1}\left\{\Psi_{k+1}\right\}^{-1}$。

## 4. 算例仿真

在本部分中，给出算例仿真来说明本文所提出的算法的有效性。

系统参数取值如下：

$$\vec{A}_k=\begin{bmatrix}0.8 & 0.5\\-0.1 & 0.6+0.03\sin\left(2k\right)\end{bmatrix},\quad\vec{B}_k=\begin{bmatrix}0.3\\0.5\end{bmatrix},$$

非线性函数选取如下：

$$\vec{f}\left(\vec{x}_k\right)=G_k\vec{x}_k+\begin{bmatrix}0.3\sin\left(\vec{x}_{1,k}+\vec{x}_{2,k}\right)\\0.1\vec{x}_{1,k}\sin\left(2k\right)\end{bmatrix},$$

其中 $\vec{x}_k = \begin{bmatrix} \vec{x}_{1,k} \\ \vec{x}_{2,k} \end{bmatrix}$， $G_k = \begin{bmatrix} 0.72 & 0.3 \\ 0.48 & 0.5 \end{bmatrix}$。

此外，其他参数的选取如下：

$\varepsilon = 0.1$， $\mu_1 = \mu_2 = 1$， $\vec{Q}_k = 0.36$， $\vec{R}_k = 0.5$， $\bar{\alpha}_k = 0.85$， $\bar{\lambda}_k = 0.65$。

在本部分仿真实验中，选取系统的状态初始值为 $x_{0|0} = [0.2 \quad 0.2]^{\mathrm{T}}$，滤波器的初始值为 $\hat{x}_{0|0} = [0.6 \quad 0.6]^{\mathrm{T}}$。系统状态的协方差矩阵的初始值为 $\bar{X}_0 = 2I_2$，估计误差协方差矩阵的初始值为 $\Sigma_{0|0} = 10I_2$。

在进行 MATLAB 算例仿真时，考虑如下两种情形：情形 I，当触发阈值 $\theta$ 取值为 0.1 时，给出系统的状态轨迹与滤波器的估计效果对比图；情形 II，当触发阈值 $\theta$ 取值为 0.7 时，给出系统的状态轨迹与滤波器的估计效果对比图，如图 1、图 2。最后，基于情形 I 和情形 II，给出系统状态与其估计误差协方差矩阵上界的关系图，如图 3。具体仿真效果图如下：

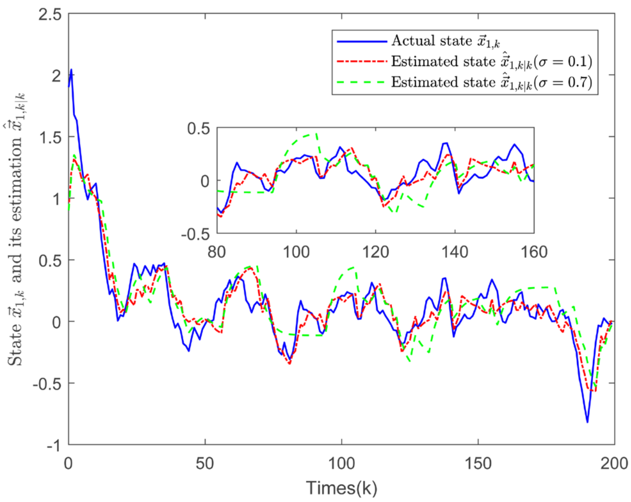


**Figure 1.** The state trajectory of $x_{1,k}$ and its estimation trajectory of $\hat{x}_{1,k|k}$

**图 1.** 实际状态轨迹 $x_{1,k}$ 及其滤波轨迹 $\hat{x}_{1,k|k}$

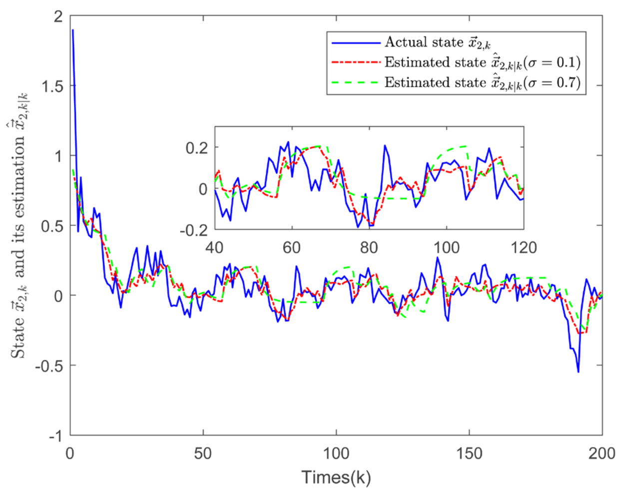


**Figure 2.** The state trajectory of $x_{2,k}$ and its estimation trajectory of $\hat{x}_{2,k|k}$

**图 2.** 实际状态轨迹 $x_{2,k}$ 及其滤波轨迹 $\hat{x}_{2,k|k}$

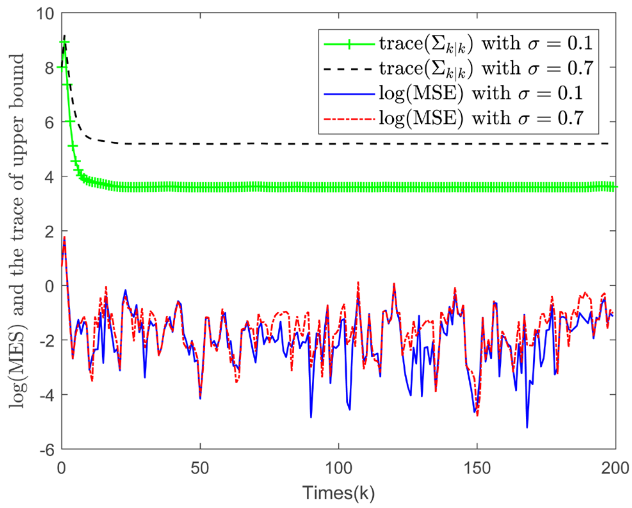


**Figure 3.** Log(MSE) and its upper bound
**图 3.** 均方误差及其上界

## 5. 结论

本文中，解决了具有一步时滞的随机发生非线性系统的滤波问题，为了刻画一步时滞与非线性的随机性，在文中引入两列服从伯努利分布的随机序列。除此之外，为了减少网络传输压力，节省网络带宽资源，引入了事件触发传输机制。通过求解类黎卡提差分方程，得到滤波误差协方差矩阵的上界，并且通过设计相应的滤波增益矩阵使得该上界的迹达到最小。

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