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Geometric Phase in a Controllable Coupling of Superconducting Flux Qubits

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Abstract

Geometric phase is an important concept in quantum mechanics. By using the Lewis-Riesenfeld invariant theory, we propose the Geometric phase in a controllable coupling of superconducting flux qubits.

Keywords

Geometric Phase, Controllable Coupling of Superconducting Flux Qubits

超导磁通量子比特的可控耦合的几何相位

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摘　要

几何相位是量子力学中的一个重要概念，通过使用Lewis-Riesenfeld不变理论，我们提出了超导磁通量子比特的可控耦合的几何相位。

关键词

几何相位，超导磁通量子比特的可控耦合

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1. 引言

1984 年 Berry 研究发现，在绝热过程中量子力学波函数存在一个不可积的具有几何性质的相位因子，它不同于通常的动力学相位因子。这一发现在许多物理领域的几何相位因子已被广泛研究和证实，并逐渐通过相关实验证实。在最近 20 多年里，几何相位因子的研究已经成为量子力学中最重要的基本问题之一，其基本概念几乎渗透到物理学的各个领域[1]-[32]。本文将通过使用 Lewis-Riesenfeld 不变理论，我们将研究超导磁通量子比特的可控耦合的几何相位。

2. 模型

根据参考文献[33]，超导通量量子位的可控耦合的有效伪螺旋哈密尔顿算子可以写为

$$\hat{H}=-\sum_{i=1,2}\left[\Delta_i\sigma_x^{(i)}+\varepsilon_i\sigma_z^{(i)}\right]+J(\phi_c)\sigma_z^{(1)}\sigma_z^{(2)}, \tag{1}$$

其中 Δ_i 是对应的隧道矩阵元素， ε_i 量子比特上的偏差，并且 σ_z^i 是磁通基中的 Pauli 矩阵，且 $\phi_c\equiv 2\pi fc$ 。我们令

$$A=-\varepsilon_1,\ B=-\Delta_1,\ C=-\varepsilon_2,\ D=-\Delta_2,\ E=J(\phi_c). \tag{2}$$

所以(1)式中的哈密顿量可以写成

$$\hat{H}=A\sigma_z^{(1)}+B\sigma_x^{(1)}+C\sigma_z^{(2)}+D\sigma_x^{(2)}+E\sigma_z^{(1)}\sigma_z^{(2)}. \tag{3}$$

其中 σ_z and σ_x 是 Pauli 矩阵中的元素且 $\sigma_+\sigma_-=2(1+\sigma_z)$ ， $\sigma_-\sigma_+=2(1-\sigma_z)$

$$[\sigma_+,\sigma_-]=\sigma_z,\ [\sigma_z,\sigma_+]=2\sigma_+,\ [\sigma_z,\sigma_-]=-2\sigma_-. \tag{4}$$

3. 动力学与几何相位

为了完整性，我们首先说明 Lewis-Riesenfeld (L-R)不变的理论[34]。对于其哈密顿量是时间相关的一维系统，则存在称为不变的算子，如果它满足等式

$$i\frac{\partial\hat{I}(t)}{\partial t}+\left[\hat{I}(t),\hat{H}(t)\right]=0. \tag{5}$$

给出了时间相关不变量 $|\lambda_n,t\rangle$ 的特征值方程

$$\hat{I}(t)|\lambda_n,t\rangle=\lambda_n|\lambda_n,t\rangle, \tag{6}$$

其中 $\frac{\partial\lambda_n}{\partial t}=0$ ，

该系统的时间相关的薛定谔方程是

$$i\frac{\partial|\psi(t)\rangle_s}{\partial t}=\hat{H}(t)|\psi(t)\rangle_s. \tag{7}$$

根据 L-R 不变理论，方程(7)的特定解|λn,t>仅与相位因子 exp [iδn(t)]不同于特征函数|λn,t>

$$|\lambda_n,t\rangle_s = \exp\left[i\delta_n(t)\right]|\lambda_n,t\rangle, \tag{8}$$

这表明$|\lambda_n,t\rangle_s\ (n=1,2,\cdots)$形成等式(7)的解的完整集合。然后薛定谔方程(7)的通解如下

$$|\psi(t)\rangle_s = \sum_n C_n \exp\left[i\delta_n(t)\right]|\lambda_n,t\rangle, \tag{9}$$

其中

$$\delta_n(t) = \int_0^t \mathrm{d}t' \langle \lambda_n,t'|i\frac{\partial}{\partial t'} - \hat{H}(t')|\lambda_n,t'\rangle,$$

$$C_n = \langle \lambda_n,0|\psi(0)\rangle_s.$$

对此系统描述的哈密顿量(3)式，我们可以定义不变量如下

$$\hat{I}(t) = \alpha_1(t)\sigma_+^{(1)} + \alpha_2(t)\sigma_+^{(2)} + \alpha_1^*(t)\sigma_-^{(1)} + \alpha_2^*(t)\sigma_-^{(2)} + \beta(t)\sigma_z^{(1)}\sigma_z^{(2)}. \tag{11}$$

代入方程(3)和(11)和(5)得到

$$\dot{\alpha}_1(t) - 2A\alpha_1(t) = 0,\ \dot{\alpha}_2(t) - 2\alpha_2(t)C = 0,\ \alpha_1(t) = \alpha_1^*(t), \tag{12}$$

$$\alpha_2(t) = \alpha_2^*(t),\ i\dot{\beta}(t) - 2E\left[\alpha_1(t) - \alpha_2(t)\right] = 0. \tag{13}$$

为了获得与时间无关的不变量，我们可以引入么正算符

$$\hat{V}(t) = \exp\left[\xi(t)\sigma_+^{(1)} - \xi^*(t)\sigma_-^{(1)}\right]\exp\left[\mu(t)\sigma_+^{(2)} - \mu^*(t)\sigma_-^{(2)}\right].$$

如果我们让非保利矩阵的第三分量的系数为零，由于有限的空间，这里不再给出这些关系，则出现时间无关的不变量

$$\hat{I}_V \equiv \hat{V}^+(t)\hat{I}(t)\hat{V}(t) = \sigma_z^{(1)}\sigma_z^{(2)}, \tag{14}$$

以下关系成立

$$\begin{aligned}&\left\{\sqrt{2|\xi|}\alpha_1\xi\left[\sin\sqrt{2|\xi|} + \frac{1}{\sqrt{2|\xi|}} - 1\right] + \sqrt{2|\xi|}\alpha_1^*\xi^*\left[\sin\sqrt{2|\xi|} + \frac{1}{\sqrt{2|\xi|}} - 1\right]\right\}\\ &\times\left\{\sqrt{2|\mu|}\alpha_2\mu\left[\sin\sqrt{2|\mu|} + \frac{1}{\sqrt{2|\mu|}} - 1\right] + \sqrt{2|\mu|}\alpha_2^*\mu^*\left[\sin\sqrt{2|\mu|} + \frac{1}{\sqrt{2|\mu|}} - 1\right]\right\}\beta\cos(2|\mu|)\cos(2|\xi|) = 1.\end{aligned} \tag{15}$$

通过使用 Baker-Campbell-Hausdoff 公式[35]

$$\hat{V}^+(t)\frac{\partial\hat{V}(t)}{\partial t} = \frac{\partial\hat{L}}{\partial t} + \frac{1}{2!}\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right] + \frac{1}{3!}\left[\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right],\hat{L}\right] + \frac{1}{4!}\left[\left[\left[\frac{\partial\hat{L}}{\partial t},\hat{L}\right],\hat{L}\right],\hat{L}\right] + \cdots, \tag{16}$$

$\hat{V}(t) = \exp\left[\hat{L}(t)\right]$，当满足以下等式时容易发现

$$A + B + \frac{\beta\xi}{|\xi|}\left[\sin(2|\xi|) + 2|\xi| - 1\right] + \frac{2\xi\left(\xi\dot{\xi}^* - \dot{\xi}\xi^*\right)}{(2|\xi|)^3}\left[\sin(2|\xi|) - 2|\xi|^3\right] = 0, \tag{17}$$

有

$$
\begin{aligned}
\hat{H}_V(t) &= \hat{V}^{+}(t)\hat{H}(t)\hat{V}(t) - i\hat{V}^{+}(t)\frac{\partial \hat{V}(t)}{\partial t} \\
&= i\left[1-\cos(2|\xi|)\right]\left(\xi\dot{\xi}^{*}-\dot{\xi}\xi^{*}\right) + i\left[1-\cos(2|\mu|)\right]\left(\mu\dot{\mu}^{*}-\dot{\mu}\mu^{*}\right) \\
&\quad - i\frac{2\xi}{(2|\xi|)^{3}}\left(\xi\dot{\xi}^{*}-\dot{\xi}\xi^{*}\right)\left[\sin(2|\xi|)-2|\xi|^{3}\right]\sigma_{+}^{(1)} + \frac{2\xi^{*}\left(\xi\dot{\xi}^{*}-\dot{\xi}\xi^{*}\right)}{(2|\xi|)^{3}}\left[\sin(2|\xi|)-2|\xi|^{3}\right]\sigma_{-}^{(1)} \\
&\quad - i\frac{2\mu}{(2|\mu|)^{3}}\left(\mu\dot{\mu}^{*}-\dot{\mu}\mu^{*}\right)\left[\sin(2|\mu|)-2|\mu|^{3}\right]\sigma_{+}^{(2)} + \frac{2\mu^{*}\left(\mu\dot{\mu}^{*}-\dot{\mu}\mu^{*}\right)}{(2|\mu|)^{3}}\left[\sin(2|\mu|)-2|\mu|^{3}\right]\sigma_{-}^{(2)} \\
&\quad + A\sigma_{+}^{(1)} + A\xi\sqrt{2|\xi|}\sigma_{z}^{(1)}\left[\sin\sqrt{2|\xi|}+\frac{1}{\sqrt{2|\xi|}}-1\right] + A\sigma_{-}^{(1)} + A\xi^{*}\sqrt{2|\xi|}\sigma_{z}^{(1)}\left[\sin\sqrt{2|\xi|}+\frac{1}{\sqrt{2|\xi|}}-1\right] \\
&\quad + C\sigma_{+}^{(2)} + C\mu\sqrt{2|\mu|}\sigma_{z}^{(2)}\left[\sin\sqrt{2|\mu|}+\frac{1}{\sqrt{2|\mu|}}-1\right] + C\sigma_{-}^{(2)} + C\mu^{*}\sqrt{2|\mu|}\sigma_{z}^{(2)}\left[\sin\sqrt{2|\mu|}+\frac{1}{\sqrt{2|\mu|}}-1\right] \\
&\quad + E\sigma_{z}^{(1)}\left\{\sigma_{z}^{(2)}\cos(2|\mu|) + \frac{\mu}{|\mu|}\left[\sin(2|\mu|)+2|\mu|-1\right]\sigma_{+}^{(2)} + \frac{\mu^{*}}{|\mu|}\left[\sin(2|\mu|)+2|\mu|-1\right]\sigma_{-}^{(2)}\right\} \\
&\quad + B\sigma_{+}^{(1)} + B\xi\sqrt{2|\xi|}\sigma_{z}^{(1)}\left[\sin\sqrt{2|\xi|}+\frac{1}{\sqrt{2|\xi|}}-1\right] + B\sigma_{-}^{(1)} + B\xi^{*}\sqrt{2|\xi|}\sigma_{z}^{(1)}\left[\sin\sqrt{2|\xi|}+\frac{1}{\sqrt{2|\xi|}}-1\right] \\
&\quad + D\sigma_{+}^{(2)} + D\mu\sqrt{2|\mu|}\sigma_{z}^{(2)}\left[\sin\sqrt{2|\mu|}+\frac{1}{\sqrt{2|\mu|}}-1\right] + D\sigma_{-}^{(2)} + D?^{*}\sqrt{2|\mu|}\sigma_{z}^{(2)}\left[\sin\sqrt{2|\mu|}+\frac{1}{\sqrt{2|\mu|}}-1\right],
\end{aligned}
\tag{18}
$$

几何相位为

$$
\dot{\delta}^{g}(t) = i\left[1-\cos(2|\xi|)\right]\left(\xi\dot{\xi}^{*}-\dot{\xi}\xi^{*}\right) + i\left[1-\cos(2|\mu|)\right]\left(\mu\dot{\mu}^{*}-\dot{\mu}\mu^{*}\right) \tag{19}
$$

显然，该几何相位是纯几何的，具有可观察效应。我们期望未来的实验能检测到。这对未来的应用有潜在的价值。

4. 结论

在本文中，我们研究了超导通量量子位的可控耦合中的几何相位。我们发现几何相位与偏置电压，隧道项矩阵元以及对称约瑟夫逊能量的耦合器无关，是纯几何效应,且具有实验上可观察性质。我们期望未来在新器件中得到检验。

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