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$1/m_Q$ Expansion for Heavy Baryons and $\Lambda_b \to \Lambda_c l\bar{\nu}$ Transition in HQEFT

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Abstract: The transition matrix elements between heavy baryons each of which contains a single heavy quark are studied in the framework of heavy quark effective field theory. The framework is derived from the full QCD and the heavy quark-antiquark coupling effects are manifestly included in the effective Lagrangian. The transition matrix elements between heavy baryons are expanded up to order $1/m_Q^2$. A set of heavy quark spin-flavor independent wave functions are defined to describe these transitions. The form factors and baryon masses are then related to these wave functions. Based on the $1/m_Q$ expansion, we discuss the semileptonic branching fraction of $\Lambda_b \to \Lambda_c l\bar{\nu}$.

Keywords: Heavy Quark Effective Field Theory; Baryon; $1/m_Q$ Expansion

重夸克有效场论下重子矩阵元的 $1/m_Q$ 展开及 $\Lambda_b \to \Lambda_c l\bar{\nu}$ 衰变

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摘　要：本文在重夸克有效场论框架下讨论了含有一个重夸克的重子间跃迁矩阵元的重夸克展开。该有效场论来自对完整 QCD 重夸克场的分解，其特点是有效拉氏密度中包括了正、反重夸克耦合项。本文将重子间跃迁矩阵元展开讨论到了 $1/m_Q^2$ 阶，定义与重夸克自旋、味无关的波函数，得到形状因子、重子质量用这些波函数表示的 $1/m_Q$ 展开式。利用这些结果，文中从重子质量估计波函数的零反冲点值，并讨论了 $\Lambda_b \to \Lambda_c l\bar{\nu}$ 的衰变分支比。

关键词：重夸克有效场论；重子；$1/m_Q$ 展开

1. 引言

本文涉及的重强子是指含有一个重夸克(b，c)和若干个轻夸克(u, d, s)的强子，包括重介子和重重子。这类强子的研究对于精确检验标准模型、深入了解强相互作用性质和探寻新物理都有重要意义。一方面，对重强子性质和衰变的研究是准确抽取标准模型重要参数如 CKM 矩阵元$|V_{cb}|$、$|V_{ub}|$的基本途径。另一方面，由于重夸克质量 m_Q 的特殊性，可能利用重夸克对称性把不同衰变联系起来，并且通过重夸克展开方法把强相互作用的微扰、非微扰效应系统地表示和研究[1-5]。

重介子结构相对简单，相应的遍举和单举衰变成为抽取$|V_{cb}|$和$|V_{ub}|$的最主要渠道。其中对$|V_{cb}|$的确定已经

很精确，而$|V_{ub}|$的准确性则差很多，特别是目前从遍举衰变得到的结果明显比从单举衰变得到结果要低[6,7]。这说明研究方法还有待完善，而对更多重强子过程的研究也是很有必要的。相对而言，对重重子的研究比重介子落后很多。对于含一个重夸克和两个轻夸克的重子，由于其中的重夸克质量远大于两个轻夸克质量及 QCD 特征能标 Λ_{QCD}，近似的重夸克对称性仍然成立，而对重夸克极限情况的修正则可按 $1/m_Q$ 展开逐级讨论，因此可以采用和介子相似的展开方法进行处理。

重夸克对称性和重夸克展开[8-10]已得到广泛应用。例如，文献[8]和[9]在通常的重夸克有效理论下分别对重介子和重重子间跃迁矩阵元作了重夸克展开。本文则是在另一种重夸克展开框架[10]下进行讨论，其特点是直接从 QCD 基本理论出发，对 QCD 基本夸克场做完整分解后仅把重夸克场小分量积分出去而得到有效拉氏密度，结果是在有效拉氏密度中包括了正、反重夸克耦合项。文献[10]中已用这种展开方法讨论了重介子半轻衰变 $B \to D(D^*) l\bar{\nu}$ 及$|V_{cb}|$的抽取，本文则把该重夸克展开框架应用于重子。第二节将对所用的展开方法作简单介绍；第三节把该展开方法应用于 $\Lambda_b \to \Lambda_c l\bar{\nu}$ 类型重子跃迁矩阵元的分析；第四节根据所得结果对 $\Lambda_b \to \Lambda_c l\bar{\nu}$ 衰变波函数和分支比作初步估算。

2. 重夸克有效场论和重子衰变矩阵元

在文献[10]中，从完整 QCD 理论出发，把完整 QCD 场分解为大、小分量，再把小分量积分出去，可以得到一个包含正、反重夸克耦合算符贡献的有效拉氏密度。这里我们称相应的有效理论为重夸克有效场论(HQEFT)以区别于通常的重夸克有效理论(HQET)[8,9]。在 HQET 中，夸克、反夸克被分开处理，即在开始导出有效拉氏密度时即认为夸克与反夸克是退耦合的。当然，两种不同的方法在领头阶是相同的，差别仅来自对正、反重夸克耦合的考虑，相应的重夸克展开中差异开始于 $1/m_Q$ 阶修正。

对于由一个重夸克和两个轻夸克组成的重子，重夸克对称性与介子类似。重夸克的四维动量可以表示为 $p_Q{}^\mu = m_Q v^\mu + k^\mu$，其中 v^μ 为重子的四维速度。残余动量 k^μ 远小于 $m_Q v^\mu$，它标志了重夸克离壳程度。设 $|H(p)\rangle$ 为完整 QCD 中的重子态，p 为重子动量，重子质量为 m_H。由矢量流守恒，重子态满足归一化条件：

$$\langle H(p)|\bar{Q}\gamma^\mu Q|H(p)\rangle = 2p^\mu = 2m_H v^\mu \tag{2.1}$$

当 $m_Q \to \infty$ 时，引入与重夸克味道无关的重子态，其归一化形式为：

$$\langle H_v|\bar{Q}_v\gamma^\mu Q_v|H_v\rangle = 2\bar{\Lambda}v^\mu \tag{2.2}$$

其中 Q_v 为有效重夸克场[10]。这里我们用 $\bar{\Lambda}_H$ 表示重子质量与重夸克质量之差 $\bar{\Lambda}_H = m_H - m_Q$，而 $\bar{\Lambda} = \lim_{m_Q\to\infty}\bar{\Lambda}_H$ 与重夸克质量无关。

这样，完整 QCD 中的态(矩阵元)与有效理论中的态(矩阵元)之间的关系为：

$$\frac{\sqrt{\bar{\Lambda}_H\bar{\Lambda}_{H'}}}{\sqrt{m_H m_{H'}}}\langle H'(p')|\bar{Q}\Gamma Q|H(p)\rangle = \langle H'_{v'}|J^{eff}_{Q,v}(x)|H_v\rangle \tag{2.3}$$

其中 Γ 可为伽玛矩阵的任意组合，等式右边为重夸克有效场论中的衰变矩阵元，$J^{eff}_{Q,v}(x)$ 为有效流，衰变矩阵元的重夸克展开需要考虑有效流和态的修正，其展开结果是(保留到 $1/\hat{m}_Q^2$ 阶)：

$$\begin{aligned}
A = \langle H'_{v'}|J^{eff}_{Q,v}(x)|H_v\rangle = \langle H'_{v'}||\overline{Q'}_{v'}\Bigg\{&\Gamma - \frac{1}{2\hat{m}_Q}\Gamma\frac{-P_+}{\bar{\Lambda}+iv\cdot D}\left(D_\perp^2+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\right) - \frac{1}{2\hat{m}_{Q'}}\left(\overleftarrow{D}_\perp^2+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\right)\frac{-P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma\\
&-\frac{1}{4\hat{m}_Q^2}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\left[\left(D_\perp^2+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\right)\left(iv\cdot D-\bar{\Lambda}\right)-iv_\alpha D_\beta F^{\alpha\beta}+v_\alpha\sigma_{\rho\nu}D^\rho F^{\nu\alpha}\right]\\
&-\frac{1}{4\hat{m}_{Q'}^2}\left[\left(-iv'\cdot\overleftarrow{D}-\bar{\Lambda}\right)\left(\overleftarrow{D}_\perp^2+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\right)+iF^{\alpha\beta}v'_\alpha\overleftarrow{D}_\beta - F^{\nu\alpha}\overleftarrow{D}^\rho v'_\alpha\sigma_{\rho\nu}\right]\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma\\
&+\frac{1}{4\hat{m}_Q^2}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\left(D_\perp^2+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\right)\frac{P_+}{\bar{\Lambda}+iv\cdot D}\left(D_\perp^2+\frac{i}{2}\sigma_{\rho\nu}F^{\rho\nu}\right)\\
&+\frac{1}{4\hat{m}_{Q'}^2}\left(\overleftarrow{D}_\perp^2+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\right)\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\left(\overleftarrow{D}_\perp^2+\frac{i}{2}\sigma_{\rho\nu}F^{\rho\nu}\right)\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma\\
&+\frac{1}{4\hat{m}_Q\hat{m}_{Q'}}\left(\overleftarrow{D}_\perp^2+\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\right)\frac{P'_+}{\bar{\Lambda}-iv'\cdot\overleftarrow{D}}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\left(D_\perp^2+\frac{i}{2}\sigma_{\rho\nu}F^{\rho\nu}\right)\Bigg\}Q_v|H_v\rangle
\end{aligned} \tag{2.4}$$

其中 $P_+ = \frac{1+\not{v}}{2}$，$P'_+ = \frac{1+\not{v}'}{2}$，$D_\perp^\alpha = D^\alpha - (v\cdot D)v^\alpha$，$\bar{D}_\alpha$ 定义为 $\int \kappa \bar{D}_\alpha \varphi = -\int \kappa D_\alpha \varphi$。注意(2.4)式实际上是采用了按照 $1/\hat{m}_Q$ 展开的形式，其中缀饰重夸克质量 $\hat{m}_Q = m_Q + \bar{\Lambda}$，(2.4)式具体推导请参考文献[10]。

3. $\Lambda_b \to \Lambda_c l\bar{\nu}$ 跃迁矩阵元的展开

考虑从一个含质量为 m_Q 的重夸克的重子 H 到一个含质量为 $m_{Q'}$ 的重子 H' 的半轻弱衰变，它由矢量流和轴矢流的强子矩阵元决定，可以定义 6 个形状因子：

$$\left\langle H'(p')\middle|\overline{Q'}\gamma^\mu\gamma^5 Q\middle|H(p)\right\rangle = \frac{\sqrt{m_H m_{H'}}}{\sqrt{\bar{\Lambda}_H\bar{\Lambda}_{H'}}}\overline{U'}(v',s) \times\left[F_1(\omega)\gamma^\mu + F_2(\omega)v^\mu + F_3(\omega)v'^\mu\right]U(v,s) \tag{3.1}$$

$$\left\langle H'(p')\middle|\overline{Q'}\gamma^\mu\gamma^5 Q\middle|H(p)\right\rangle = \frac{\sqrt{m_H m_{H'}}}{\sqrt{\bar{\Lambda}_H\bar{\Lambda}_{H'}}}\overline{U'}(v',s) \times\left[G_1(\omega)\gamma^\mu + G_2(\omega)v^\mu + G_3(\omega)v'^\mu\right]\gamma^5 U(v,s) \tag{3.2}$$

其中旋量 U 满足归一化条件

$$\bar{U}(v,s)\gamma^\mu U(v,s) = 2\bar{\Lambda}v^\mu \tag{3.3}$$

及 $\not{v}U = U \quad \not{v}'\overline{U'} = \bar{U}$。

另一方面，式子(3.1)、(3.2)左边的矩阵元可以按照(2.3)、(2.4)式作逐级重夸克展开。对于所得展开式中的每项，可以参数化为与重夸克质量、自旋无关的波函数。例如对于(2.4)式中领头阶，有

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\Gamma Q_v\middle|H_v\right\rangle = \xi(\omega)\overline{U'}\Gamma U$$

Isgur-Wise 函数 $\xi(\omega)$ 只与反冲值 $\omega = v\cdot v'$ 有关。对于 $1/\hat{m}_Q$ 修正的各项，可以作类似的参数化。考虑到 $1/\hat{m}_Q^2$ 阶，如下定义波函数：

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\Gamma\frac{-P_+}{\bar{\Lambda}+iv\cdot D}D_\perp^2 Q_v\middle|H_v\right\rangle = -\kappa_1(\omega)\frac{1}{\bar{\Lambda}}\overline{U'}\Gamma U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\bar{D}_\perp^2\frac{-P'_+}{\bar{\Lambda}-iv'\cdot\bar{D}}\Gamma Q_v\middle|H_v\right\rangle = -\kappa_1(\omega)\frac{1}{\bar{\Lambda}}\overline{U'}\Gamma U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\Gamma\frac{-P_+}{\bar{\Lambda}+iv\cdot D}\left[D_\perp^2\left(iv\cdot D-\bar{\Lambda}\right)-iv_\alpha D_\beta F^{\alpha\beta}\right]Q_v\middle|H_v\right\rangle = -\rho_1(\omega)\frac{1}{\bar{\Lambda}}\overline{U'}\Gamma U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\left[\left(-iv'\cdot\bar{D}-\bar{\Lambda}\right)\bar{D}_\perp^2 + iF^{\alpha\beta}v'_\alpha\bar{D}_\beta\right]\frac{P'_+}{\bar{\Lambda}-iv'\cdot\bar{D}}\Gamma Q_v\middle|H_v\right\rangle = -\rho_1(\omega)\frac{1}{\bar{\Lambda}}\overline{U'}\Gamma U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}D_\perp^2\frac{P_+}{\bar{\Lambda}+iv\cdot D}D_\perp^2 Q_v\middle|H_v\right\rangle = \chi_1(\omega)\frac{1}{\bar{\Lambda}}\overline{U'}\Gamma U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\bar{D}_\perp^2\frac{P'_+}{\bar{\Lambda}-iv'\cdot\bar{D}}\bar{D}_\perp^2\frac{P'_+}{\bar{\Lambda}-iv'\cdot\bar{D}}\Gamma Q_v\middle|H_v\right\rangle = \chi_1(\omega)\frac{1}{\bar{\Lambda}}\overline{U'}\Gamma U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{P_+}{\bar{\Lambda}+iv\cdot D}\frac{i}{2}\sigma_{\rho\nu}F^{\rho\nu}Q_v\middle|H_v\right\rangle = \chi^{\alpha\beta\rho\nu}(v,v')\frac{1}{\bar{\Lambda}^2}\overline{U'}\Gamma P_+\frac{i}{2}\sigma_{\alpha\beta}P_+\frac{i}{2}\sigma_{\rho\nu}U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\frac{i}{2}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{P'_+}{\bar{\Lambda}-iv'\cdot\bar{D}}\frac{i}{2}\sigma_{\rho\nu}F^{\rho\nu}\frac{P'_+}{\bar{\Lambda}-iv'\cdot\bar{D}}\Gamma Q_v\middle|H_v\right\rangle = \chi^{\alpha\beta\rho\nu}(v',v)\frac{1}{\bar{\Lambda}^2}\overline{U'}\frac{i}{2}\sigma_{\alpha\beta}P'_+\frac{i}{2}\sigma_{\rho\nu}P'_+\Gamma U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\bar{D}_\perp^2\frac{P'_+}{\bar{\Lambda}-iv'\cdot\bar{D}}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}D_\perp^2 Q_v\middle|H_v\right\rangle = -\frac{1}{\bar{\Lambda}^2}\eta_1(\omega)\overline{U'}\Gamma U$$

$$\left\langle H'_{v'}\middle|\overline{Q'}_{v'}\sigma_{\alpha\beta}F^{\alpha\beta}\frac{P'_+}{\bar{\Lambda}-iv'\cdot\bar{D}}\Gamma\frac{P_+}{\bar{\Lambda}+iv\cdot D}\sigma_{\rho\nu}F^{\rho\nu}Q_v\middle|H_v\right\rangle = -\eta^{\alpha\beta\rho\nu}(v,v')\overline{U'}\sigma_{\alpha\beta}P'_+\Gamma P_+\sigma_{\gamma\delta}U$$

其中

$$\chi^{\alpha\beta\rho\nu}(v,v') = \chi^{\rho\nu\alpha\beta}(v,v') = \chi_2(\omega)\left(g^{\alpha\rho}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\rho}\right) + \chi_3(\omega)\left(g^{\alpha\rho}v'^\beta v'^\nu - g^{\beta\rho}v'^\alpha v'^\nu - g^{\alpha\nu}v'^\beta v'^\rho + g^{\beta\nu}v'^\alpha v'^\rho\right)$$

$$\eta^{\alpha\beta\rho\nu}(v,v') = \eta_2(\omega)\left(g^{\alpha\rho}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\rho}\right) + \eta_3(\omega)\left(g^{\alpha\rho}v^\beta v'^\nu - g^{\beta\rho}v^\alpha v'^\nu - g^{\alpha\nu}v^\beta v'^\rho + g^{\beta\nu}v^\alpha v'^\rho\right)$$

用以上定义的波函数表示，对弱衰变 $\Lambda_b \to \Lambda_c l\bar{\nu}$ 而言(2.4)式中的矩阵元展开可简化为：

$$
\begin{aligned}
A=&\left[\xi(\omega)+\frac{1}{2\bar{\Lambda}}\left(\frac{1}{\hat{m}_Q}+\frac{1}{\hat{m}_{Q'}}\right)\kappa_1(\omega)+\left(\frac{1}{\hat{m}_Q{}^2}+\frac{1}{\hat{m}_{Q'}^2}\right)\left(\frac{1}{4\bar{\Lambda}^2}\chi_1(\omega)+\frac{1}{4\bar{\Lambda}}\rho_1(\omega)\right)-\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\eta_1(\omega)\right]\overline{U'}\Gamma U\\
&+\frac{-1}{16\hat{m}_Q^2}\frac{1}{\bar{\Lambda}^2}\overline{U}'\Gamma\left(2\chi_2(\omega)P_+\sigma_{\rho\nu}P_+\sigma^{\rho\nu}+4\chi_3(\omega)P_+\sigma_{\rho\beta}P_+\sigma^{\rho\nu}v'^{\beta}v'_{\nu}\right)U\\
&+\frac{-1}{16\hat{m}_Q^2}\frac{1}{\bar{\Lambda}^2}\overline{U'}\left(2\chi_2(\omega)\sigma^{\rho\nu}P'_+\sigma_{\rho\nu}P'_++4\chi_3(\omega)\sigma_{\rho\beta}P'_+\sigma^{\rho\nu}P'_+v^{\beta}v_{\nu}\right)\Gamma U\\
&+\frac{1}{16\hat{m}_Q\hat{m}_{Q'}}\frac{1}{\bar{\Lambda}^2}\overline{U'}\left(2\eta_2(\omega)\sigma_{\rho\nu}P'_+\Gamma P_+\sigma^{\rho\nu}+4\eta_3(\omega)\sigma_{\rho\beta}P'_+\Gamma P_+\sigma^{\rho\nu}v^{\beta}v'_{\nu}\right)U
\end{aligned}
\tag{3.4}
$$

当 $\Gamma=\gamma^{\mu}$，(3.4)式成为：

$$
\begin{aligned}
A_{(\Gamma=\gamma^{\mu})}=&\left[\xi(\omega)+\frac{1}{2\bar{\Lambda}}\left(\frac{1}{\hat{m}_Q}+\frac{1}{\hat{m}_{Q'}}\right)\kappa_1(\omega)+\left(\frac{1}{\hat{m}_Q^2}+\frac{1}{\hat{m}_{Q'}^2}\right)\right.\\
&\times\left(\frac{1}{4\bar{\Lambda}}\rho_1(\omega)+\frac{1}{4\bar{\Lambda}^2}\chi_1(\omega)-\frac{3}{4\bar{\Lambda}^2}\chi_2(\omega)-\frac{1}{2\bar{\Lambda}^2}\chi_3(\omega)\left(1-\omega^2\right)\right)\\
&\left.-\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left(\eta_1(\omega)+\eta_2(\omega)\right)\right]\overline{U'}\gamma^{\mu}U+\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left[2\eta_2(\omega)+2(\omega-1)\eta_3(\omega)\right]\overline{U'}\left(v^{\mu}+v'^{\mu}\right)U
\end{aligned}
$$

在零反冲点 $\omega=1$，并假设末态与初态相同，即 $H'=H$，利用(2.1)~(2.3)及(3.3)可得：

$$
\begin{aligned}
\bar{\Lambda}_H=&\bar{\Lambda}+\frac{1}{\hat{m}_Q}\kappa_1(1)+\frac{1}{2}\frac{1}{\hat{m}_Q{}^2}\rho_1(1)\\
&+\frac{1}{4\bar{\Lambda}}\frac{1}{\hat{m}_Q{}^2}\left(2\chi_1(1)-\eta_1(1)-6\chi_2(1)+3\eta_2(1)\right)
\end{aligned}
\tag{3.5}
$$

由于 $\bar{\Lambda}_H=m_H-m_Q$，上式给出了重子质量与波函数零反冲点值的关系。

对于 H' 与 H 为不同重子的情形，根据(2.3)式和(3.4)式可得到 QCD 矩阵元的展开结果。对矢量流和轴矢流部分，分别有：

$$
\begin{aligned}
A_v\equiv&\left\langle H'(p')\right|\bar{Q}\gamma^{\mu}Q\left|H(p)\right\rangle=\frac{\sqrt{m_Hm_{H'}}}{\sqrt{\bar{\Lambda}_H\bar{\Lambda}_{H'}}}\left\{\left[\xi(\omega)+\frac{1}{2\bar{\Lambda}}\left(\frac{1}{\hat{m}_Q}+\frac{1}{\hat{m}_{Q'}}\right)\kappa_1(\omega)\right.\right.\\
&+\left(\frac{1}{\hat{m}_Q{}^2}+\frac{1}{\hat{m}_{Q'}^2}\right)\left(\frac{1}{4\bar{\Lambda}}\rho_1(\omega)+\frac{1}{4\bar{\Lambda}^2}\chi_1(\omega)-\frac{3}{4\bar{\Lambda}^2}\chi_2(\omega)-\frac{1}{2\bar{\Lambda}^2}\chi_3(\omega)\left(1-\omega^2\right)\right)\\
&\left.\left.-\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left(\eta_1(\omega)+\eta_2(\omega)\right)\right]\overline{U'}\gamma^{\mu}U+\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left[2\eta_2(\omega)+2(\omega-1)\eta_3(\omega)\right]\overline{U'}\left(v^{\mu}+v'^{\mu}\right)U\right\}
\end{aligned}
$$

和

$$
\begin{aligned}
A_{av}\equiv&\left\langle H'(p')\right|\bar{Q}\gamma^{\mu}\gamma^5Q\left|H(p)\right\rangle=\frac{\sqrt{m_Hm_{H'}}}{\sqrt{\bar{\Lambda}_H\bar{\Lambda}_{H'}}}\left\{\left[\xi(\omega)+\frac{1}{2\bar{\Lambda}}\left(\frac{1}{\hat{m}_Q}+\frac{1}{\hat{m}_{Q'}}\right)\kappa_1(\omega)+\frac{1}{4\bar{\Lambda}}\left(\frac{1}{\hat{m}_Q{}^2}+\frac{1}{\hat{m}_{Q'}^2}\right)\rho_1(\omega)\right.\right.\\
&\left.+\frac{1}{4\bar{\Lambda}^2}\left(\frac{1}{\hat{m}_Q{}^2}+\frac{1}{\hat{m}_{Q'}^2}\right)\left(\chi_1(\omega)-3\chi_2(\omega)-2\chi_3(\omega)\left(1-\omega^2\right)\right)-\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left(\eta_1(\omega)+\eta_2(\omega)\right)\right]\overline{U'}\gamma^{\mu}\gamma^5U\\
&\left.+\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left[2\eta_2(\omega)+2(\omega+1)\eta_3(\omega)\right]\overline{U'}\left(v'^{\mu}-v^{\mu}\right)U\right\}
\end{aligned}
$$

与形状因子的定义式(3.1)、(3.2)相比较，可得形状因子用波函数表示的展开式：

$$F_1(\omega)=\xi(\omega)+\frac{1}{2\bar{\Lambda}}\left(\frac{1}{\hat{m}_Q}+\frac{1}{\hat{m}_{Q'}}\right)\kappa_1(\omega)$$
$$+\left(\frac{1}{\hat{m}_Q{}^2}+\frac{1}{\hat{m}_{Q'}^2}\right)\left(\frac{1}{4\bar{\Lambda}}\rho_1(\omega)+\frac{1}{4\bar{\Lambda}^2}\chi_1(\omega)-\frac{3}{4\bar{\Lambda}^2}\chi_2(\omega)-\frac{1}{2\bar{\Lambda}^2}\chi_3(\omega)\left(1-\omega^2\right)\right) \quad (3.6)$$
$$-\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left(\eta_1(\omega)+\eta_2(\omega)\right)$$

$$F_2(\omega)=F_3(\omega)=\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left[2\eta_2(\omega)+2(\omega-1)\eta_3(\omega)\right] \quad (3.7)$$

$$G_1(\omega)=F_1(\omega) \quad (3.8)$$

$$G_2(\omega)=-G_3(\omega)=\frac{-1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left[2\eta_2(\omega)+2(\omega+1)\eta_3(\omega)\right] \quad (3.9)$$

在零反冲点处有：

$$F_1(1)=1+\frac{1}{2\bar{\Lambda}}\left(\frac{1}{\hat{m}_Q}+\frac{1}{\hat{m}_{Q'}}\right)\kappa_1(1)$$
$$+\left(\frac{1}{\hat{m}_Q{}^2}+\frac{1}{\hat{m}_{Q'}{}^2}\right)\left(\frac{1}{4\bar{\Lambda}}\rho_1(1)+\frac{1}{4\bar{\Lambda}^2}\chi_1(1)-\frac{3}{4\bar{\Lambda}^2}\chi_2(1)\right)$$
$$-\frac{1}{4\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left(\eta_1(1)+\eta_2(1)\right) \quad (3.10)$$

$$F_2(1)=F_3(1)=\frac{1}{2\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\eta_2(1) \quad (3.11)$$

$$G_1(1)=F_1(1) \quad (3.12)$$

$$G_2(1)=-G_3(1)=\frac{-1}{2\bar{\Lambda}^2}\frac{1}{\hat{m}_Q\hat{m}_{Q'}}\left[\eta_2(1)+2\eta_3(1)\right] \quad (3.13)$$

4. 波函数与衰变分支比

形状因子或波函数表征了强子衰变的完整信息，对它们的详细计算需要采用格点模拟或 QCD 求和规则等非微扰方法。但利用上节得到的形状因子、波函数和强子质量的关系，可以对 $\Lambda_b \to \Lambda_c l\bar{\nu}$ 衰变作现象学的初步讨论。

在只考虑到 $1/\hat{m}_Q$ 阶修正的情况下，由(3.4)式有：

$$m_H=m_Q+\bar{\Lambda}_H=m_Q+\bar{\Lambda}+\frac{1}{m_Q+\bar{\Lambda}}\kappa_1(1) \quad (4.1)$$

对于重子 Λ_b (udb)和 Λ_c (udc)，质量 $m(\Lambda_b)=5.619$ GeV，$m(\Lambda_c)=2.286$ GeV [11]。夸克质量有较大不确定性，文献[10]在同样的有效场论框架下讨论半轻衰变 $B\to D(D^*)l\bar{\nu}$，从重介子质量估算重介子跃迁波函数。讨论中对介子而言 m_b-m_c (或 $\hat{m}_b-\hat{m}_c$)所取较合理范围为3.45~3.55 GeV。若取夸克质量 $m_b-m_c=3.50$ GeV，$m_b=4.65$ GeV [11]则由(4.1)式可得

$$\bar{\Lambda}\approx 0.89\ \text{GeV} \qquad \kappa_1(1)\approx 0.53\ \text{GeV}^2$$

由(3.10)、(3.12)式得到

$$F_1(1)=G_1(1)=1+\frac{1}{2\bar{\Lambda}}\left(\frac{1}{\hat{m}_b}+\frac{1}{\hat{m}_c}\right)\kappa_1(1)+O\left(\frac{1}{\hat{m}_Q^2}\right)\approx 1.21 \quad (4.2)$$

可见 $1/\hat{m}_Q$ 阶波函数 $\kappa_1(1)$ 对形状因子有约20%的修正。

忽略轻子质量，$\Lambda_b \to \Lambda_c l\bar{\nu}$ 的微分衰变率可表示为[12]：

$$\frac{1}{\sqrt{\omega^2-1}}\frac{\mathrm{d}\Gamma\left(\Lambda_b\to\Lambda_c l\bar{\nu}\right)}{\mathrm{d}\omega}=\frac{G_F^2\left|V_{cb}\right|^2}{(2\pi)^3}m_{\Lambda_c}^3m_{\Lambda_b}^2$$
$$\times\left(\left(1-2r\omega+r^2\right)\left[(\omega-1)F_1^{*2}+(\omega+1)G_1^{*2}\right]\right. \quad (4.3)$$
$$\left.+\frac{\omega^2-1}{3}\left(Ar^2+2Br+C\right)\right)$$

其中 $r=m_{\Lambda_c}/m_{\Lambda_b}$，

$$A=2F_1^*F_2^*+(\omega+1)F_2^{*2}+2G_1^*G_2^*+(\omega-1)G_2^{*2}$$

$$B=F_1^{*2}+F_1^*F_2^*+F_2^*F_3^*+F_3^*F_1^*+\omega F_2^*F_3^*+G_1^{*2}-G_1^*G_2^*-G_2^*G_3^*+G_3^*G_1^*+\omega G_2^*G_3^*$$

$$C = (\omega+1)F_3^{*2} + 2F_1^* F_3^* + (\omega-1)G_3^{*2} - 2G_1^* G_3^*$$

根据定义，上列式子中 $F_i^*(\omega)$ 和 $G_i^*(\omega)$ 与本文中的 $F_i(\omega)$ 和 $G_i(\omega)$ 相差一个因子 $\dfrac{\bar{\Lambda}}{\sqrt{\bar{\Lambda}_{\Lambda_b}\bar{\Lambda}_{\Lambda_c}}}$，即

$$F_i^*(\omega) = \frac{\bar{\Lambda}}{\sqrt{\bar{\Lambda}_{\Lambda_b}\bar{\Lambda}_{\Lambda_c}}} F_i(\omega) \quad (i=1,2,3)$$

$$G_i^*(\omega) = \frac{\bar{\Lambda}}{\sqrt{\bar{\Lambda}_{\Lambda_b}\bar{\Lambda}_{\Lambda_c}}} G_i(\omega) \quad (i=1,2,3)$$

在忽略 $1/\hat{m}_Q^2$ 阶修正的情况下(4.3)式简化为：

$$\frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c l\bar{\nu})}{\mathrm{d}\omega} = \frac{\bar{\Lambda}^2}{\bar{\Lambda}_{\Lambda_b}\bar{\Lambda}_{\Lambda_c}} \frac{G_F^2 |V_{cb}|^2}{4\pi^3} m_{\Lambda_c}^3 m_{\Lambda_b}^2 \sqrt{\omega^2-1} \times \left[\omega(1-2r\omega+r^2) + \frac{2}{3} r(\omega^2-1)\right] F_1^2(\omega) \tag{4.4}$$

领头阶 Isgur-Wise 函数随 ω 的变化已经被用 QCD 求和规则[12]、相对论夸克模型[13,14]、Skyrme 模型[15]、bag 模型[16]等方法作了研究，而高阶波函数还未被系统地详细讨论。这里我们假设 F_1 随 ω 的变化方式与 $\xi(\omega)$ 类似，且满足最简单的关系[17]，

$$F_1(\omega) = F_1(1)\mathrm{e}^{-\hat{\rho}^2(\omega-1)} \tag{4.5}$$

其中 $\hat{\rho}^2 = 2.0_{-1.1}^{+0.8}$ [14,17]。如果取 $\hat{\rho}^2 = 2.0$，$|V_{cb}| = 4.09\times10^{-2}$，则(4.4)式积分可得到总衰变宽度

$$\Gamma \approx 2.38\times10^{-14}\ \text{GeV}$$

结合 Λ_b 的寿命值 $(1.425\pm0.032)\times10^{-12}$ s [11]可得衰变分支比：

$$Br(\Lambda_b \to \Lambda_c l\bar{\nu}) \approx 5.14\%$$

但如果不考虑 $F_1(\omega)$ 中的 $1/\hat{m}_Q$ 修正，结果为：

$$Br(\Lambda_b \to \Lambda_c l\bar{\nu}) \approx 3.51\%$$

这里的主要不确定性来源于 $\hat{\rho}^2$ 和夸克质量。如果取 $\hat{\rho}^2 = 2.0_{-1.1}^{+0.8}$ [14,17]，$m_b - m_c = (3.50\pm0.05)$ GeV，$m_b = (4.65\pm0.03)$ GeV，可得到

$$Br(\Lambda_b \to \Lambda_c l\bar{\nu}) = \left(5.14_{-1.63}^{+3.56}\right)\%$$

图 1 和图 2 分别显示了分支比随 $\hat{\rho}^2$ 及质量差 $m_b - m_c$ 的变化情况。

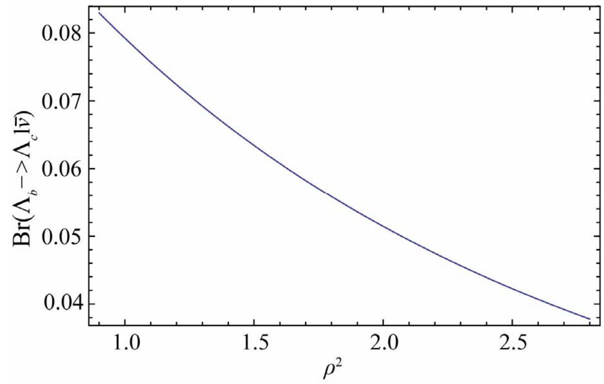

Figure 1. Cruve: relation between $Br(\Lambda_b \to \Lambda_c l\bar{\nu})$ and $\hat{\rho}^2$ when $m_b = 4.65$ GeV and $m_b - m_c = 3.50$ GeV

图 1. 当 $m_b = 4.65$ GeV，$m_b - m_c = 3.50$ GeV 时，$Br(\Lambda_b \to \Lambda_c l\bar{\nu})$ 随 $\hat{\rho}^2$ 变化情况

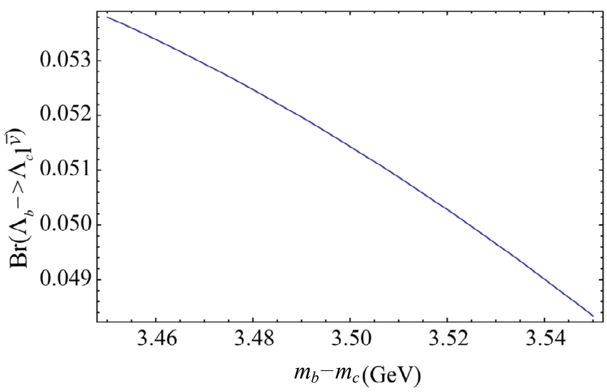

Figure 2. Cruve: relation between $Br(\Lambda_b \to \Lambda_c l\bar{\nu})$ and $m_b - m_c$ when $m_b = 4.65$ GeV and $\hat{\rho}^2 = 2.0$

图 2. 当 $m_b = 4.65$ GeV，$\hat{\rho}^2 = 2.0$ 时，$Br(\Lambda_b \to \Lambda_c l\bar{\nu})$ 随 $m_b - m_c$ 变化情况

5. 小结

本文对重重子跃迁矩阵元作了重夸克有效场论下的展开分析。定义了与重夸克自旋、味无关的波函数，得到了形状因子间一些关系，例如 $F_1(\omega) = G_1(\omega)$，且 $F_2(\omega)$、$F_3(\omega)$ 与 $G_2(\omega)$、$G_3(\omega)$ 直到 $1/\hat{m}_Q$ 阶均为零。另外，重子质量可与波函数零反冲点值联系起来。文中从重子质量估计了 $1/\hat{m}_Q$ 阶的波函数，其对形状因子展开领头阶的修正大约为 20%。估算 $\Lambda_b \to \Lambda_c l\bar{\nu}$ 衰变的分支比为 $Br(\Lambda_b \to \Lambda_c l\bar{\nu}) = \left(5.14_{-1.63}^{+3.56}\right)\%$。从估算结果看，对该衰变的精确讨论需要认真考虑 $1/\hat{m}_Q$ 修正的贡献。

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