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Lump Solution for Sawada-Kotera-Kadovtsev Petviashvili Equation

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Abstract

Using the symbolic calculation software Mathematics and the Hirota bilinear operator, the lump solutions of the Sawada-Kotera-Kadomtsev-Petviashvili equation are discussed. We have obtained 7-case lump solutions. We choose one-kind lump solution of them. Its 3D graphics and contour maps are given, when the parameters included in the lump solution take special values. From those graphics, one can observe the characteristics of this lump solution with the increase of time *t*.

Keywords

Lump Solution, Sawada-Kotera-Kadomtsev-Petviashvili Equation

Sawada-Kotera-Kadovtsev-Petviashvili方程的Lump解

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摘　要

利用符号计算软件Mathematics和Hirota双线性算子，研究了Sawada-Kotera-Kadomtsev-Petviashvili方程的lump解。我们得到了该方程的7类lump解，选取一类lump解，当参数取特值时，给出了不同的*t*值对应的3D图形和等高线图。由此可以观察到这个lump解随时间*t*的增加而变化的特性。

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关键词

Lump解，Sawada-Kotera-Kadomtsev-Petviashvili方程

1. 引言

许多物理和力学现象都是非线性的。例如，1895 年 D. J. Korteweg 和 G. de Vries [1]发现水波现象是非线性的，并给出了著名的 Korteweg-de Vries (KdV)方程；Von Kárman [2]首次指出薄板挠度问题是非线性问题，并且 1940 年给出了该问题的控制方程——著名的 Kárman 方程式。因此求解非线性微分方程成为应用数学领域的一个热点问题。迄今为止，许多研究者提出并发展了求解非线性微分方程的各种方法，如 Lie 对称[3]、Hirota 双线性法[4] [5] [6]、同伦摄动法[7] [8]、同伦分析法[9]、Adomian 分解法[10] [11] (ADM)、辅助方程法[12] [13] [14]、Bäcklund 变换和 Hirota 双线性法的结合[15] [16] [17] [18] [19]等。

其中，Hirota 双线性方法是日本著名数学、物理学家 Ryogo Hirota [4]提出的一种求解非线性微分方程的方法。近年来，通过 Hirota 双线性方法，研究了 lump 解与反应解。例如：YTSF 方程的 lump 解与反应解[20]，(2 + 1)维非线性发展方程的反应解[21]，(3 + 1)维非线性发展方程的精确解[22]，降维双线性方程的 lump 解[23]，广义(3 + 1)维 Kadomtsev-Petviashvili 方程的 lump 解[24]；非局部 KPI 方程的 lump 解[25]；新的广义 Kadomtsev-Petviashvili 方程的 lump 解[26]；2 + 1 维 Korteweg-de-Vries 方程的个一般 lump 解[27]；Boiti-Leon-Manna-Pempinelli (BLMP)方程的 lump 解[28]；广义变系数 Kadomtsev-Petviashvili 方程的 lump 解[29]；Ito 方程的 lump 解[30]；Boussinesq 波方程的 lump 单孤子解和 lump 双孤子解[31]；(3 + 1)维 BKP-Boussinesq 方程及其降维方程的 lump 解[32]；(3 + 1)维广义 B 型 Kadomtsev-Petviashvili 方程的 lump 解、lump 孤子解和 lump 条孤子解[33]；Sawada-Kotera (SK)方程的 lump 解[34]；三阶非线性发展方程的 lump 解和反应解[35]；(2 + 1)维非对称 Nizhnik-Novikov-Veselov 方程 lump 解和 lump 条状解[36]等等。

用 Hirota 双线性方法从一个非线性微分方程出发，构造一个具有物理意义的非线性微分方程也是可能的。基于 Hirota 双线性方法，Wazwaz [37]结合 Sawada-Kotera (SK)方程与 Kadomtsev-Petviashvili (KP)方程提出了 Sawada-Kotera-Kadomtsev-Petviashvili (SKKP)方程。Sawada-Kotera-Kadomtsev Petviashvili (SK-KP)方程如下：

$$\left(u_t+15uu_{xxx}+15u_xu_{xx}+45u^2u_x+u_{xxxxx}\right)_x+u_{yy}=0 \tag{1}$$

在变换 $u=2\left(\ln f\right)_{xx}$ 下 SK-KP 方程变为以下 Hirota 双线性形式：

$$\begin{aligned}&B_{\text{SK-KP}}\left(f\right):=\left(D_xD_t+D_x^6+D_y^2\right)f\cdot f\\&=2\left(-f_y^2-f_tf_x-10f_{xxx}^2+15f_{xx}f_{xxxx}-6f_xf_{xxxxx}\right)+f\left(f_{yy}+f_xf_t+f_{xxxxxx}\right)\\&=0\end{aligned} \tag{2}$$

其中双线性算子如下：$D_t^nD_x^ma\cdot b=\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^n\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^m a\left(x,t\right)\cdot b\left(x',t'\right)\Big|_{x'=x,t'=t}$。

基于双线性方程(2)，Wazwaz [37]讨论了它的 n-孤子解；Wazwaz 利用 tanh-coth 方法[38]讨论了其单孤子解；在[39] [40]中，利用 F 展开法和指数函数法讨论了 SK-KP 方程的精确行波解、孤立波解、新的扭波解和周期波解。

基于上述变换，本文利用 Hirota 双线性方法和符号计算系统 Mathematics 将研究 SK-KP 方程(1)的 lump 解。具体内容如下：第二节在 Hirota 双线性方法的基础上，讨论了 SK-KP 方程的 lump 解，得到了七类 lump 解；第三节给出了结论。

2. SK-KP 方程的 Lump 解

为了讨论(1)中的 Lump 解，我们假设方程(2)中的 f 为：

$$f=g^2+h^2+a_9, \tag{3}$$

其中，$g=a_1x+a_2y+a_3t+a_4$，$h=a_5x+a_6y+a_7t+a_8$，$a_i\left(i=1,2,\cdots,9\right)$ 都是实数。将(3)代入式(2)，得到了关于 $a_i\left(i=1,2,\cdots,9\right)$ 的代数方程组。在求解该代数方程组，得到以下七组解和相应的方程(1)的 Lump 解：

第一组：

$$a_1=a_1,a_2=0,a_3=0,a_4=a_4,a_5=a_5,a_6=0,a_7=0,a_8=a_8,a_9=a_9 \tag{4}$$

其中 $a_i\left(i=1,4,5,8,9\right)$ 是任意实数，对应 SK-KP 方程的 lump 解如下：

$$u\left(x,y,t\right)=\frac{p}{q} \tag{5}$$

这里 $p=2\left(2\left(a_1^2+a_5^2\right)\left(\left(a_1x+a_4\right)^2+\left(a_5x+a_8\right)^2+a_9\right)-4\left(a_1^2x+a_5\left(a_5x+a_8\right)+a_4a_1\right)^2\right)$，

$$q=\left(\left(a_1x+a_4\right)^2+\left(a_5x+a_8\right)^2+a_9\right)^2.$$

第二组： 当 $a_7\neq 0$ 时，

$$a_1=0,a_2=0,a_3=0,a_4=a_4,a_5=-\frac{a_6^2}{a_7},a_6=a_6,a_7=a_7,a_8=a_8,a_9=a_9 \tag{6}$$

相应方程的解如下：

$$u\left(x,y,t\right)=\frac{4a_6^2\left(-\left(a_7t-\frac{a_6^2x}{a_7}+a_6y+a_8\right)^2+a_4^2+a_9\right)}{a_7^2\left(\left(a_7t-\frac{a_6^2x}{a_7}+a_6y+a_8\right)^2+a_4^2+a_9\right)}. \tag{7}$$

第三组：

$$a_1=a_1,a_2=0,a_3=a_3,a_4=a_4,a_5=-\frac{a_1a_7}{a_3},a_6=-\sqrt{a_1}\sqrt{\frac{a_7^2}{a_3}+a_3},a_7=a_7,a_8=a_8,a_9=0 \tag{8}$$

其中 $a_3\neq 0,a_1\geq 0$。相应的 lump 解为：

$$u\left(x,y,t\right)=\frac{p}{q} \tag{9}$$

这里

$$p=4a_1^2\left[a_3^2\left(a_3^2+a_7^2\right)\left(\left(xa_1+ta_3+a_4\right)^2+\frac{\left(xa_1a_7+y\sqrt{a_1}a_3\sqrt{a_3+\frac{a_7^2}{a_3}}-a_3\left(ta_7+a_8\right)\right)^2}{a_3^2}\right)\right.,$$

$$\left.-2\left(xa_1\left(a_3^2+a_7^2\right)+y\sqrt{a_1}a_3a_7\sqrt{a_3+\frac{a_7^2}{a_3}}+a_3\left(ta_3^2+a_3a_4-a_7\left(ta_7+a_8\right)\right)\right)^2\right]$$

$$q=a_3^4\left(\left(xa_1+ta_3+a_4\right)^2+\frac{\left(xa_1a_7+y\sqrt{a_1}a_3\sqrt{a_3+\frac{a_7^2}{a_3}}-a_3\left(ta_7+a_8\right)\right)^2}{a_3^2}\right)^2.$$

第四组： 当 $a_3\neq 0, a_1\geq 0$ 时，

$$a_1=a_1, a_2=0, a_3=a_3, a_4=a_4, a_5=-\frac{a_1a_7}{a_3}, a_6=\sqrt{a_1}\sqrt{\frac{a_7^2}{a_3}+a_3}, a_7=a_7, a_8=a_8, a_9=0$$

方程的解为：

$$u(x,y,t)=\frac{p}{q}$$

这里

$$p=4a_1^2\left[a_3^2\left(a_3^2+a_7^2\right)\left(\left(xa_1+ta_3+a_4\right)^2+\frac{\left(-xa_1a_7+y\sqrt{a_1}a_3\sqrt{a_3+\frac{a_7^2}{a_3}}+a_3\left(ta_7+a_8\right)\right)^2}{a_3^2}\right)\right.,$$

$$\left.-2\left(xa_1\left(a_3^2+a_7^2\right)-y\sqrt{a_1}a_3a_7\sqrt{a_3+\frac{a_7^2}{a_3}}+a_3\left(ta_3^2+a_3a_4-a_7\left(ta_7+a_8\right)\right)\right)^2\right]$$

$$q=a_3^4\left(\left(xa_1+ta_3+a_4\right)^2+\frac{\left(-xa_1a_7+y\sqrt{a_1}a_3\sqrt{a_3+\frac{a_7^2}{a_3}}+a_3\left(ta_7+a_8\right)\right)^2}{a_3^2}\right)^2.$$

第五组： 当 $a_6\neq 0$ 和 $a_6\neq 0$ 时，

$$a_1=-\frac{a_2a_6}{a_7},a_2=a_2,a_3=\frac{a_2a_7}{a_6},a_4=a_4,a_5=-\frac{a_6^2}{a_7},a_6=a_6,a_7=a_7,a_8=a_8,a_9=a_9$$

方程的解为：

$$u(x,y,t)=\frac{p}{q}$$

其中，

$$\begin{aligned}p=-4\Big[&2a_2^3a_4a_6a_7\left(-xa_6^2+ya_6a_7+ta_7^2\right)+a_2^4\left(-xa_6^2+ya_6a_7+ta_7^2\right)^2\\&+2a_2a_4a_6^3a_7\left(-xa_6^2+ya_6a_7+a_7\left(ta_7+2a_8\right)\right)+a_2^2a_6^2\left(2x^2a_6^4-4xya_6^3a_7\right.\\&\left.+2ya_6a_7^2\left(2ta_7+a_8\right)-2a_6^2a_7\left(\left(2tx-y^2\right)a_7+xa_8\right)+a_7^2\left(a_4^2+2t^2a_7^2+2ta_7a_8-a_8^2-a_9\right)\right),\\&+a_6^4\left(x^2a_6^4-2xya_6^3a_7+2ya_6a_7^2\left(ta_7+a_8\right)+a_6^2a_7\left(\left(-2tx+y^2\right)a_7-2xa_8\right)\right.\\&\left.+a_7^2\left(-a_4^2+t^2a_7^2+2ta_7a_8+a_8^2-a_9\right)\right)\Big]\end{aligned}$$

$$q=a_7^4\left(\frac{\left(a_4a_6a_7+a_2\left(-xa_6^2+ya_6a_7+ta_7^2\right)\right)^2}{a_6^2a_7^2}+\left(ya_6-\frac{xa_6^2}{a_7}+ta_7+a_8\right)^2+a_9\right)^2.$$

第六组：当 $a_3\neq0$ 时，

$$a_1=-\frac{a_2^2}{a_3},a_2=a_2,a_3=a_3,a_4=a_4,a_5=0,a_6=0,a_7=0,a_8=a_8,a_9=a_9$$

方程的解为：

$$u(x,y,t)=\frac{4a_2^2\left(-\left(a_3t-\frac{a_2^2x}{a_3}+a_2y+a_4\right)^2+a_8^2+a_9\right)}{a_3^2\left(\left(a_3t-\frac{a_2^2x}{a_3}+a_2y+a_4\right)^2+a_8^2+a_9\right)^2}.$$

第七组：当 $a_3^2+a_7^2\neq0$ 时，

$$a_1=\frac{-a_3a_2^2-2a_6a_7a_2+a_3a_6^2}{a_3^2+a_7^2},a_2=a_2,a_3=a_3,a_4=a_4,$$
$$a_5=\frac{a_7a_2^2-2a_3a_6a_2-a_7a_6^2}{a_3^2+a_7^2},a_6=a_6,a_7=a_7,a_8=a_8,a_9=0$$

方程的解为：

$$u(x,y,t)=\frac{p}{q},$$

其中，

$$
\begin{aligned}
p = 4\Big\{&-2\Big[-xa_2^4 + ya_2^3a_3 + a_2^2\left(ta_3^2 + a_3a_4 - 2xa_6^2 + ya_6a_7 - a_7\left(ta_7 + a_8\right)\right)\\
&+ a_6^2\left(-ta_3^2 - a_3a_4 - xa_6^2 + ya_6a_7 + a_7\left(ta_7 + a_8\right)\right) + a_2a_6\left(2a_4a_7 + a_3\left(ya_6 + 4ta_7 + 2a_8\right)\right)\Big]^2\\
&+\left(a_2^2 + a_6^2\right)^2\Big[\Big(x^2a_2^4 - 2xya_2^3a_3 + t^2a_3^4 + 2ta_3^3a_4 + x^2a_6^4 - 2xya_6^3a_7 + a_4^2a_7^2 - 2tx\,a_6^2a_7^2\\
&+ y^2a_6^2a_7^2 + 2tya_6a_7^3 + t^2a_7^4 + 2a_3a_4\left(xa_6^2 + ta_7^2\right) - 2xa_6^2a_7a_8 + 2ya_6a_7^2a_8 + 2ta_7^3a_8\\
&+ a_7^2a_8^2 + a_3^2\left(a_4^2 + \left(2tx + y^2\right)a_6^2 + 2t^2a_7^2 + 2ta_7a_8 + a_8^2 + 2ya_6\left(ta_7 + a_8\right)\right)\\
&+ a_2^2\left(\left(-2tx + y^2\right)a_3^2 - 2xa_3a_4 + 2x^2a_6^2 - 2xya_6a_7 + a_7\left(\left(2tx + y^2\right)a_7 + 2xa_8\right)\right)\\
&+ 2a_2\left(tya_3^3 + ya_3^2a_4 + a_4a_7\left(-2xa_6 + ya_7\right) - a_3\left(xya_6^2 - tya_7^2 + 2xa_6\left(2ta_7 + a_8\right)\right)\right)\Big)\Big]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
q = \left(a_3^2 + a_7^2\right)^2\Bigg[&\frac{\left(-xa_2^2a_3 + ta_3^3 + a_3^2a_4 + a_4a_7^2 + a_3\left(xa_6^2 + ta_7^2\right) + a_2\left(ya_3^2 + a_7\left(-2xa_6 + ya_7\right)\right)\right)^2}{\left(a_3^2 + a_7^2\right)^2}\\
&+\left(ya_6 + ta_7 + \frac{x\left(-2a_2a_3a_6 + a_2^2a_7 - a_6^2a_7\right)}{a_3^2 + a_7^2} + a_8\right)^2\Bigg].
\end{aligned}
$$

3. 结论

本文借助于符号计算系统 Mathematics 和双线性算子，研究了 SKKP 方程的 Lump 解。我们得到了 7 组 lump 解。由此说明 SKKP 方程的解的丰富性和部分解的特征以便理解该方程的可积性。当 $a_1 = 4, a_3 = 3, a_4 = 2, a_7 = 0, a_8 = 3$ 时，解(9)的 3D 图和等高线图如图 1 所示，可以观察到随着时间该 lump 解沿着 x 轴向正方向移动。

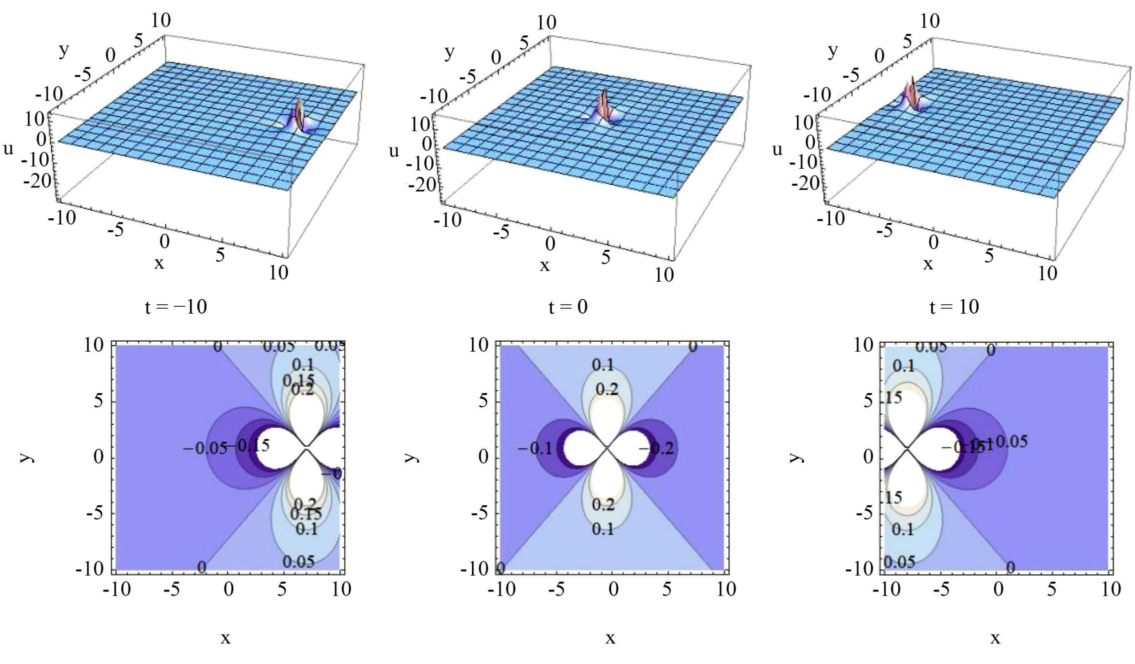

Figure 1. 3D plot and contour plot of solution (9)
图 1. 解(9)的 3D 图和等高线图($a_1 = 4, a_3 = 3, a_4 = 2, a_7 = 0, a_8 = 3$)

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