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概率双层语言T球形模糊集及其在多属性决策中的应用

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摘 要

作为模糊集扩展之一的T球形模糊集，由于引入了参数q，使其能够在更大范围内处理不确定信息。相比以往的语言术语集，双层语言术语集在表达决策者意见时，具有优越性。文章在考虑决策者犹豫心理的情况下，提出概率双层语言T球形模糊集的新概念，定义其基本运算、Hamming距离以及排序规则。基于Dombi运算规则，构建加权平均算子，并对其性质进行研究。结合信任度和Shapley值确定决策者的综合权重，利用IDOCRIW方法得到属性的客观权重。进而，提出基于概率双层语言T球形模糊集的多属性决策方法。最后将所提决策框架应用于一个实际案例，并通过对比分析，验证所提方法的有效性。

关键词

概率双层语言，T球形模糊集，Dombi范数，聚合算子，多属性决策

Probabilistic Double Hierarchy Linguistic T-Spherical Fuzzy Set and Its Application in Multi-Attribute Decision Making

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Abstract

As one of the extensions of fuzzy sets, T-spherical fuzzy sets can deal with uncertain information in

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a wider range due to the introduction of parameter *q*. Compared with the previous language term sets, the double hierarchy linguistic term set has advantages in expressing the opinions of decision-makers. Therefore, considering the hesitation of decision makers, the paper proposes a new concept of probabilistic double hierarchy linguistic T-spherical fuzzy sets and defines its basic operation, Hamming distance and ranking rules. Based on the Dombi operation rules, the weighted average operator is proposed, and its properties are investigated. The comprehensive weight of the decision maker is determined by combining the trust degree and the Shapley value, and the objective weight of the attribute is obtained by using the IDOCRIW method. Furthermore, a multi-attribute decision-making method based on probabilistic double hierarchy linguistic T-spherical fuzzy sets is investigated. Finally, the proposed decision-making framework is applied to a practical case, and the effectiveness of the proposed method is verified by comparative analysis.

Keywords

Probabilistic Double Linguistic, T-Spherical Fuzzy Set, Dombi Norm, Aggregation Operator, Multi-Attribute Decision Making

1. 引言

多属性决策方法一直是学术界的一个研究热点[1] [2]。现实世界问题常存在亦此亦彼的模糊性现象，这促使了模糊集理论的产生，该理论为解决多属性决策问题提供了新的途径。美国控制论专家 Zadeh [3]首次提出模糊集的概念，其核心思想在于构造取值于区间[0, 1]的隶属度函数，以刻画亦此亦彼的连续过渡状态。Atanassov 等[4]提出直觉模糊集，扩展了模糊集的隶属度函数，用隶属度和非隶属度二元结构表示不确定信息。随后，学者们进一步提出毕达哥拉斯模糊集[5]、q 阶正交模糊集[6]，使得模糊集在实际应用范围更广。考虑到决策者的犹豫心理或者多个决策者的评价值不完全统一，Torra 等[7]引入犹豫模糊集(Hesitant Fuzzy Set, HFS)。与模糊集相比，HFS 允许决策者用多个隶属度表达。之后，Coung 等[8]又提出图模糊集，将直觉模糊集的二元结构拓展到三元结构，但其无法刻画三类隶属度之和大于 1 的情形。Mahmood 等[9]为拓宽图模糊集的范围，提出球形模糊集和 T 球形模糊集。而 T 球形模糊集由于在约束条件中引入了参数 q，在处理不确定性时更为优越。

传统的模糊集可以处理不确定信息，但只能从定量的角度来表征，因此 Zadeh [3]引入语言变量处理语言评价信息。自语言变量提出以来，已有许多关于语言型多准则决策问题的研究[10] [11]。然而，在决策过程中有时可能需要用一些更复杂的语言术语来表示决策者的意见，如“非常好”、“不太差”。为了解决这个问题，Gou 等[12]提出双层语言术语集，该语言术语集包含两层语言术语集，因此能够更为准确地表达决策者的意见。犹豫模糊集的每个隶属度取值的重要程度是相同的，但在实际生活中，这可能违背了决策者真正的想法。决策者更倾向于用一个概率分布来反映各个隶属度的重要性。比如 Pang 等[13]在不损失决策者所有原始语言信息的前提下，通过增加概率扩展了 HFSs，提出概率语言术语集。Gou 等[14]考虑在双层犹豫语言术语集中引入概率信息，定义概率双层语言模糊集。Shafiq 等[15]对 T 球形模糊集进行拓展，提出概率语言 T 球形模糊集。

聚合算子是将多元信息组合成单一形式的工具，在多属性决策问题中发挥着重要作用。在众多聚合算子中，加权几何算子是应用最为广泛的算子之一，它们都是基于某些 t-范数和 t-余范数提出的。近来，

基于 Dombi 范数的聚合算子受到众多学者关注，如 Mandal 等[16]利用基于 Dombi 范数运算规则融合权重信息聚合区间球型模糊数据信息。

结合以上分析，本文利用双层语言术语能够更为清晰准确地表达决策者意见，结合 T 球形模糊集结构，同时考虑决策者的犹豫性，提出概率双层语言 T 球形模糊集，并且定义其得分函数、精确函数以及 Hamming 距离；提出基于 Dombi 范数加权平均算子，并讨论该算子的性质。在多属性决策问题中，决策者以及属性的权重在聚合过程中对结果有重要影响。针对决策者的权重确定方法中，信任度作为一种主观确定权重的方法之一，忽略了客观信息。因此，本文结合 Shapley 值与信任度确定决策者综合权重。然后，通过 IDOCRIW 方法确定属性权重，构建一个概率双层语言 T 球形模糊集多属性决策框架，并将其应用于实际案例。

2. 预备知识

定义 1 [12]设 $S=\{s_t \mid t=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ 是第一层语言术语集，$O=\{o_k \mid k=-\varsigma,\cdots,-1,0,1,\cdots,\varsigma\}$ 是第二层语言术语集，双层语言术语集定义为

$$S_O=\left\{s_{t\langle o_k\rangle} \mid t=-\tau,\cdots,-1,0,1,\cdots,\tau;k=-\varsigma,\cdots,-1,0,1,\cdots,\varsigma\right\} \tag{1}$$

其中 $s_{t\langle o_k\rangle}$ 为双层语言术语。

定义 2 [12]设 $S_O=\left\{s_{t\langle o_k\rangle} \mid t\in[-\tau,\tau];k\in[-\varsigma,\varsigma]\right\}$ 为连续的双层语言术语集，任意一个双层术语 $s_{t\langle o_k\rangle}$ 与之对应的隶属度 γ 的转化函数定义为如下

$$f:[-\tau,\tau]\times[-\varsigma,\varsigma]\to[0,1],\ f\left(s_{t\langle o_k\rangle}\right)=\frac{k+(\tau+t)\varsigma}{2\tau}=\gamma \tag{2}$$

$$f^{-1}:[0,1]\to[-\tau,\tau]\times[-\varsigma,\varsigma],\ f^{-1}(\gamma)=s_{[2\tau\gamma-\tau]+1\left\langle o_{\varsigma(2\tau\gamma-\tau-[2\tau\gamma-\tau])-\varsigma}\right\rangle} \tag{3}$$

定义 3 [12]设 X 是一个有限论域，S_o 是一个双层语言术语集，定义在 X 上的概率双层语言模糊集表示为

$$P_s=\left\{x_i \middle| z(p)(x_i) \middle| x_i\in X\right\} \tag{4}$$

其中 $z(p)(x_i)$ 称为概率双层语言元，其定义式为

$$z(p)(x_i)=\left\{z^\sigma\left(p^\sigma\right) \mid z^\sigma\in S_O,p^\sigma\in[0,1],\sigma=1,2,\cdots,L,\sum_{\sigma=1}^{L}p^\sigma\le 1\right\} \tag{5}$$

其中 L 表示双层语言术语集的个数，p^σ 表示对应语言术语的概率。

定义 4 [9]设 X 是一个有限论域，则 T 球形模糊集可定义为

$$T=\left\{\left\langle x,\mu_T(x),\eta_T(x),\nu_T(x)\right\rangle \mid x\in X\right\} \tag{6}$$

其中，μ_T,ν_T,π_T 为 X 到[0, 1]的一个映射，分别表示元素 x 属于 T 的积极隶属度、中立隶属度和消极隶属度，且满足 $0\le\mu_T^q+\eta_T^q+\nu_T^q\le 1$，$q$ 为正整数。称 $\pi_T=\sqrt[q]{1-\mu_T^q-\eta_T^q-\nu_T^q}$ 为元素 x 对 T 球形模糊集 T 的拒绝度。

定义 5 [17]设 $(x,y)\in[0,1]\times[0,1],\theta>0$，Dombi 的 t-norm 和 t-conorm 分别定义如下

$$TN(x,y)=\frac{1}{1+\left\{\left(\frac{1-x}{x}\right)^\theta+\left(\frac{1-y}{y}\right)^\theta\right\}^{\frac{1}{\theta}}} \tag{7}$$

$$TCN(x,y)=1-\frac{1}{1+\left\{\left(\frac{x}{1-x}\right)^{\theta}+\left(\frac{y}{1-y}\right)^{\theta}\right\}^{\frac{1}{\theta}}} \tag{8}$$

3. 概率双层语言 T 球形模糊集

定义 6 设 X 是一个有限论域，S_0 是一个双层语言术语集，定义在 X 上的概率双层语言 T 球形模糊集(Probabilistic Double Hierarchy Linguistic T-Spherical Fuzzy Sets, PDHLTSFS)表示为

$$Y(p)=\left\{\left\langle x,H_s(p)(x),G_s(p)(x),I_s(p)(x)\right\rangle \mid x\in X\right\} \tag{9}$$

其中，$H_s(p)(x)=\left\{\varphi^r\left(p_\varphi^r\right)\mid \varphi^r\in S_O,p_\varphi^r\in[0,1],r=1,2,\cdots,L_1,\sum_{r=1}^{L_1}p_\varphi^r\le 1\right\}$ 表示 x 对 $Y(P)$ 的积极隶属度，$G_s(p)(x)=\left\{\phi^d\left(p_\phi^d\right)\mid \phi^d\in S_O,p_\phi^d\in[0,1],d=1,2,\cdots,L_2,\sum_{d=1}^{L_2}p_\phi^d\le 1\right\}$ 表示 x 对 $Y(P)$ 的中立隶属度，$I_s(p)=\left\{\psi^m\left(p_\psi^m\right)\mid \psi^m\in S_O,p_\psi^m\in[0,1],m=1,2,\cdots,L_3,\sum_{m=1}^{L_3}p_\psi^m\le 1\right\}$ 表示 x 对 $Y(P)$ 的消极隶属度。对于任意 $x\in X$，满足 $0\le\left(\alpha^+\right)^q+\left(\beta^+\right)^q+\left(\chi^+\right)^q\le 1$，其中 $\alpha^+=\max_r\left\{f\left(\varphi^r\right)\right\}$，$\beta^+=\max_d\left\{f\left(\phi^d\right)\right\}$，$\chi^+=\max_m\left\{f\left(\psi^m\right)\right\}$，$q$ 为正整数。

当 $q=2,\varsigma=0$ 时，概率双层语言 T 球形模糊集退化为概率语言 T 球形模糊集。且当 $L_1=L_2=L_3=1$，$p_\varphi^r=p_\psi^r=p_\phi^r=1$ 时，概率语言 T 球形模糊集退化为语言图模糊集。当论域中 X 只有一个元素 x 时，称 $a=\left\langle H_s(p),G_s(p),I_s(p)\right\rangle=\left\langle\left\{\varphi^r\left(p_\varphi^r\right)\right\},\left\{\phi^d\left(p_\phi^d\right)\right\},\left\{\psi^m\left(p_\psi^m\right)\right\}\right\rangle$ 为概率双层语言 T 球形模糊元(PDHLTSFE)。记论域 X 上的所有模糊元组成的集合为 $\Gamma_{(\tau,\varsigma)}^X$。

设概率双层语言 T 球形模糊元 $a=\left\langle\left\{\varphi_a^r\left(p_{\varphi_a}^r\right)\right\},\left\{\phi_a^d\left(p_{\phi a}^d\right)\right\},\left\{\psi_a^m\left(p_{\psi_a}^m\right)\right\}\right\rangle$，$b=\left\langle\left\{\varphi_b^r\left(p_{\varphi_b}^r\right)\right\},\left\{\phi_b^d\left(p_{\phi_b}^d\right)\right\},\left\{\psi_b^m\left(p_{\psi_b}^m\right)\right\}\right\rangle$，$L_1^a,L_2^a,L_3^a$ 和 L_1^b,L_2^b,L_3^b 分别为 a,b 各部分的元素个数。若 L_1^a,L_2^a,L_3^a 和 L_1^b,L_2^b,L_3^b 不完全相等，令 $\max\left\{L_1^a,L_2^a,L_3^a,L_1^b,L_2^b,L_3^b\right\}=L$，当 a,b 各部分元素个数少于 L 时，则对应部分增加概率为 0 且是语言术语最小的概率双层语言术语，直到各部分长度相等且都为 L，记长度标准化后的 a,b 为 $\hat{a}=\left\langle\left\{\varphi_a^{(\kappa)}\left(p_{\varphi_a}^{(\kappa)}\right)\right\},\left\{\phi_a^{(\kappa)}\left(p_{\phi_b}^{(\kappa)}\right)\right\},\left\{\psi_a^{(\kappa)}\left(p_{\psi_a}^{(\kappa)}\right)\right\}\right\rangle$，$\hat{b}=\left\langle\left\{\varphi_b^{(\kappa)}\left(p_{\varphi_b}^{(\kappa)}\right)\right\},\left\{\phi_b^{(\kappa)}\left(p_{\phi_b}^{(\kappa)}\right)\right\},\left\{\psi_b^{(\kappa)}\left(p_{\psi_b}^{(\kappa)}\right)\right\}\right\rangle$。

然后将模糊元 $\hat{a}$ 的各部分按照 $f\left(\varphi_a^{(\kappa)}\right)\cdot p_{\varphi_a}^{(\kappa)},f\left(\phi_a^{(\kappa)}\right)\cdot p_{\phi_a}^{(\kappa)},f\left(\psi_a^{(\kappa)}\right)\cdot p_{\psi_a}^{(\kappa)}$ （$\kappa=1,2,\cdots,L$）的值进行升序排列，$\hat{b}$ 同理，则规范化的模糊元表示为 $\dot{a}=\left\langle\left\{s_{\gamma_a}^{(\kappa)}\left(p_{\gamma_a}^{(\kappa)}\right)\right\},\left\{s_{\eta_a}^{(\kappa)}\left(p_{\eta_a}^{(\kappa)}\right)\right\},\left\{s_{\delta_a}^{(\kappa)}\left(p_{\delta_a}^{(\kappa)}\right)\right\}\right\rangle$，$\dot{b}=\left\langle\left\{s_{\gamma_b}^{(\kappa)}\left(p_{\gamma_b}^{(\kappa)}\right)\right\},\left\{s_{\eta_b}^{(\kappa)}\left(p_{\eta_b}^{(\kappa)}\right)\right\},\left\{s_{\delta_b}^{(\kappa)}\left(p_{\delta_b}^{(\kappa)}\right)\right\}\right\rangle$，$\kappa=1,2,\cdots,L$。

定义 7 设 $h(p)=\left\{\zeta_\tau\left(p_\tau\right)\mid\tau\in S_0;\sum_{\tau=1}^{t}p_\tau\le 1\right\}$ 为概率双层语言模糊元，定义概率双层语言模糊元的综合犹豫度为

$$U(h(p))=2-\frac{4}{2+\left(1-\frac{1}{t}\right)+\left(1-\sum_{\tau=1}^{t}p_\tau\right)} \tag{10}$$

易知$U\left(h(p)\right)\in[0,1]$，其中$1-\frac{1}{t}$表示$z(p)$的犹豫度，$L$越大，表示犹豫度越高；$1-\sum_{i=1}^{t}p_{\tau}$表示概率不完全性，其值越大，表示概率信息缺失越大，则犹豫度越高。

定义 8 设$a=\langle H_s(p),G_s(p),I_s(p)\rangle$为概率双层语言 T 球形模糊元，定义概率双层语言 T 球形模糊元a的得分函数为

$$\begin{aligned}S(a)=&\left(1-U\left(H_s(p)\right)\right)\sum_{r=1}^{L_1}\left(f\left(\varphi^r\right)p_{\varphi}^r\right)^q-\left(1-U\left(G_s(p)\right)\right)\sum_{d=1}^{L_2}\left(f\left(\phi^d\right)p_{\phi}^d\right)^q\\&-\left(1-U\left(I_s(p)\right)\right)\sum_{m=1}^{L_3}\left(f\left(\psi^m\right)p_{\psi}^m\right)^q\end{aligned}\tag{11}$$

定义 9 设$a=\langle H_s(p),G_s(p),I_s(p)\rangle$为概率双层语言 T 球形模糊元，定义概率双层语言 T 球形模糊元a的精确函数为

$$H(a)=\sum_{r=1}^{L_1}\left(f\left(\varphi^r\right)p_{\varphi}^r\right)^q+\sum_{d=1}^{L_2}\left(f\left(\phi^d\right)p_{\phi}^d\right)^q+\sum_{m=1}^{L_3}\left(f\left(\psi^m\right)p_{\psi}^m\right)^q\tag{12}$$

定义 10 设$a_1=\langle H_{s1}(p),G_{s1}(p),I_{s1}(p)\rangle,b_1=\langle H_{s2}(p),G_{s2}(p),I_{s2}(p)\rangle$为任意两个概率双层语言 T 球形模糊元，则它们的比较规则如下

1) 若$S(a_1)>S(b_1)$，则称a_1优于b_1，记为$a_1\succ b_1$；

2) 若$S(a_1)=S(b_1)$并且$H(a_1)>H(b_1)$，则称a_1优于b_1，记为$a_1\succ b_1$。

注：本文的运算都是在标准化和规范化后处理的基础上进行的。

定义 11 设两个规范化后的概率双层语言 T 球形模糊元分别为$a_1=\langle H_{s1}(p),G_{s1}(p),\ I_{s1}(p)\rangle$，$b_1=\langle H_{s2}(p),G_{s2}(p),I_{s2}(p)\rangle$，定义它们之间的 Hamming 距离为

$$\begin{aligned}d(a_1,b_1)=\sum_{\kappa=1}^{L}\Bigg[&p_{\gamma1}^{(\kappa)}p_{\gamma1}^{(\kappa)}\left|f\left(s_{\gamma1}^{(\kappa)}\right)^q-f\left(s_{\gamma2}^{(\kappa)}\right)^q\right|+p_{\eta1}^{(\kappa)}p_{\eta2}^{(\kappa)}\left|f\left(s_{\eta1}^{(\kappa)}\right)^q-f\left(s_{\eta1}^{(\kappa)}\right)^q\right|\\&+p_{\delta1}^{(\kappa)}p_{\delta1}^{(\kappa)}\left|f\left(s_{\delta1}^{(\kappa)}\right)^q-f\left(s_{\delta1}^{(\kappa)}\right)^q\right|\Bigg]\end{aligned}\tag{13}$$

定义 12 设S_O是一个双层语言术语集，$a=\langle H_s(p),G_s(p),I_s(p)\rangle$，$a_1=\{H_{s1}(p),G_{s1}(p),\ I_{s1}(p)\}$，$b_1=\langle H_{s2}(p),G_{s2}(p),I_{s2}(p)\rangle$为论域上的三个概率双层语言 T 球形模糊元。给定$\lambda>0$，它们基本运算定义如下：

$$neg(a)=\langle I_s(p),G_s(p),H_s(p)\rangle$$

$$\begin{aligned}a_1\oplus b_1=\Bigg\langle&\left\{f^{-1}\left(\sqrt[q]{f\left(s_{\gamma1}^{(\kappa)}\right)^q+f\left(s_{\gamma2}^{(\kappa)}\right)^q-f\left(s_{\gamma1}^{(\kappa)}\right)^q f\left(s_{\gamma2}^{(\kappa)}\right)^q}\right)\left(p_{\gamma1}^{(\kappa)}p_{\gamma2}^{(\kappa)}\right)\right\},\\&\left\{f^{-1}\left(f\left(s_{\eta1}^{(\kappa)}\right)f\left(s_{\eta2}^{(\kappa)}\right)\right)\right\}\left(p_{\eta1}^{(\kappa)}p_{\eta2}^{(\kappa)}\right),\left\{f^{-1}\left(f\left(s_{\delta1}^{(\kappa)}\right)f\left(s_{\delta2}^{(\kappa)}\right)\right)\right\}\left(p_{\delta1}^{(\kappa)}p_{\delta2}^{(\kappa)}\right)\Bigg\rangle\end{aligned}$$

$$\begin{aligned}a_1\otimes b_1=\Bigg\langle&\left\{f^{-1}\left(f\left(s_{\gamma1}^{(\kappa)}\right)f\left(s_{\gamma2}^{(\kappa)}\right)\right)\left(p_{\gamma1}^{(\kappa)}p_{\gamma2}^{(\kappa)}\right)\right\},\left\{f^{-1}\left(\sqrt[q]{f\left(s_{\eta1}^{(\kappa)}\right)^q+f\left(s_{\eta1}^{(\kappa)}\right)^q-f\left(s_{\eta1}^{(\kappa)}\right)^q f\left(s_{\eta2}^{(\kappa)}\right)^q}\right)\left(p_{\eta1}^{(\kappa)}p_{\eta2}^{(\kappa)}\right)\right\},\\&\left\{f^{-1}\left(\sqrt[q]{f\left(s_{\delta1}^{(\kappa)}\right)^q+f\left(s_{\delta2}^{(\kappa)}\right)^q-f\left(s_{\delta1}^{(\kappa)}\right)^q f\left(s_{\delta2}^{(\kappa)}\right)^q}\right)\left(p_{\delta1}^{(\kappa)}p_{\delta2}^{(\kappa)}\right)\right\}\Bigg\rangle\end{aligned}$$

$$\lambda a=\left\langle \left\{ f^{-1}\left(\sqrt[q]{\left(1-\left(1-f\left(s_{\gamma}^{(\kappa)}\right)^{q}\right)^{\lambda}\right)}\right)\left(p_{\gamma}^{(\kappa)}\right)\right\},\left\{ f^{-1}\left(f\left(s_{\eta}^{(\kappa)}\right)^{\lambda}\right)\left(p_{\eta}^{(\kappa)}\right)\right\},\left\{ f^{-1}\left(f\left(s_{\delta}^{(\kappa)}\right)^{\lambda}\right)\left(p_{\delta}^{(\kappa)}\right)\right\} \right\rangle$$

$$a^{\lambda}=\left\langle \left\{ f^{-1}\left(f\left(s_{\gamma}^{(\kappa)}\right)^{\lambda}\right)\left(p_{\gamma}^{(\kappa)}\right)\right\},\left\{ f^{-1}\left(\sqrt[q]{\left(1-\left(1-f\left(s_{\eta}^{(\kappa)}\right)^{q}\right)^{\lambda}\right)}\right)\left(p_{\eta}^{(\kappa)}\right)\right\},\left\{ f^{-1}\left(\sqrt[q]{\left(1-\left(1-f\left(s_{\delta}^{(\kappa)}\right)^{q}\right)^{\lambda}\right)}\right)\left(p_{\delta}^{(\kappa)}\right)\right\} \right\rangle$$

定理 1 设 a,a_1,b_1 为论域 X 上的任意 3 个概率双层语言 T 球形模糊元，给定 $\lambda,\lambda_1,\lambda_2\geq 0$，则它们的运算满足以下性质：

1) $a_1\oplus b_1=b_1\oplus a_1$， $a_1\otimes b_1=b_1\otimes a_1$；

2) $\left(a_1\oplus b_1\right)\oplus a=\left(a_1\oplus\left(b_1\oplus a\right)\right.$， $\left(a_1\otimes b_1\right)\otimes a=a_1\otimes\left(b_1\otimes a\right)$；

3) $\lambda\left(a_1\oplus b_1\right)=\lambda a_1\oplus\lambda b_1$， $\lambda_1 a\oplus\lambda_2 a=\left(\lambda_1+\lambda_2\right)a$；

4) $\left(a_1\otimes b_1\right)^{\lambda}=a_1^{\lambda}\otimes b_1^{\lambda}$， $a^{\lambda_1}\otimes a^{\lambda_2}=a^{(\lambda_1+\lambda_2)}$。

定义 13 设 $a=\left\{H_{s1}(p),G_{s1}(p),I_{s1}(p)\right\}$， $b=\left\langle H_{s2}(p),G_{s2}(p),I_{s2}(p)\right\rangle$ 为两个概率双层语言 T 球形模糊元，定义运算如下

$$a\underset{D}{\oplus}b=\left\langle \left\{ f^{-1}\left(\left(1-\frac{1}{1+\left(\left(\frac{f\left(s_{\gamma1}^{(\kappa)}\right)^{q}}{1-f\left(s_{\gamma1}^{(\kappa)}\right)^{q}}\right)^{\theta}+\left(\frac{f\left(s_{\gamma2}^{(\kappa)}\right)^{q}}{1-f\left(s_{\gamma2}^{(\kappa)}\right)^{q}}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\overline{p_{\gamma1}^{(\kappa)}+p_{\gamma2}^{(\kappa)}}\right)\right\},\right.$$

$$\left\{ f^{-1}\left(\left(\frac{1}{1+\left(\left(\frac{1-f\left(s_{\eta1}^{(\kappa)}\right)^{q}}{f\left(s_{\eta1}^{(\kappa)}\right)^{q}}\right)^{\theta}+\left(\frac{1-f\left(s_{\eta2}^{(\kappa)}\right)^{q}}{f\left(s_{\eta2}^{(\kappa)}\right)^{q}}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\overline{p_{\eta1}^{(\kappa)}+p_{\eta2}^{(\kappa)}}\right)\right\},$$

$$\left. \left\{ f^{-1}\left(\left(\frac{1}{1+\left(\left(\frac{f\left(s_{\gamma1}^{(\kappa)}\right)^{q}}{1-f\left(s_{\gamma1}^{(\kappa)}\right)^{q}}\right)^{\theta}+\left(\frac{f\left(s_{\gamma2}^{(\kappa)}\right)^{q}}{1-f\left(s_{\gamma2}^{(\kappa)}\right)^{q}}\right)^{\theta}\right)^{\frac{1}{\theta}}}-\frac{1}{1+\left(\left(\frac{f\left(s_{\delta1}^{(\kappa)}\right)^{q}+f\left(s_{\gamma1}^{(\kappa)}\right)^{q}}{1-f\left(s_{\delta1}^{(\kappa)}\right)^{q}-f\left(s_{\gamma1}^{(\kappa)}\right)^{q}}\right)^{\theta}+\left(\frac{f\left(s_{\delta2}^{(\kappa)}\right)^{q}+f\left(s_{\gamma2}^{(\kappa)}\right)^{q}}{1-f\left(s_{\delta2}^{(\kappa)}\right)^{q}-f\left(s_{\gamma2}^{(\kappa)}\right)^{q}}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\overline{p_{\delta1}^{(\kappa)}+p_{\delta2}^{(\kappa)}}\right)\right\}\right\rangle$$

$$a \underset{D}{\otimes} b = \left\langle \left\{ f^{-1}\left(\left(\frac{1}{1+\left(\left(\frac{1-f\left(s_{\gamma 1}^{(\kappa)}\right)^q}{f\left(s_{\gamma 1}^{(\kappa)}\right)^q} \right)^{\theta} + \left(\frac{1-f\left(s_{\gamma 2}^{(\kappa)}\right)^q}{f\left(s_{\gamma 2}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{1/\theta}} \right)^{\frac{1}{q}} \right) \left(\overline{p_{\gamma 1}^{(\kappa)} + p_{\gamma 2}^{(\kappa)}} \right) \right\}, \right.$$

$$\left\{ f^{-1}\left(\left(1-\frac{1}{1+\left(\left(\frac{f\left(s_{\eta 1}^{(\kappa)}\right)^q}{1-f\left(s_{\eta 1}^{(\kappa)}\right)^q} \right)^{\theta} + \left(\frac{f\left(s_{\eta 12}^{(\kappa)}\right)^q}{1-f\left(s_{\eta 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{1/\theta}} \right)^{\frac{1}{q}} \right) \left(\overline{p_{\eta 1}^{(\kappa)} + p_{\eta 2}^{(\kappa)}} \right) \right\},$$

$$\left\{ f^{-1}\left(\left(\frac{1}{1+\left(\left(\frac{f\left(s_{\delta 1}^{(\kappa)}\right)^q}{1-f\left(s_{\delta 1}^{(\kappa)}\right)^q} \right)^{\theta} + \left(\frac{f\left(s_{\delta 2}^{(\kappa)}\right)^q}{1-f\left(s_{\delta 2}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{1/\theta}} \right. \right. \right.$$

$$\left. \left. \left. -\frac{1}{1+\left(\left(\frac{f\left(s_{\gamma 1}^{(\kappa)}\right)^q + f\left(s_{\delta 1}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma 1}^{(\kappa)}\right)^q - f\left(s_{\delta 1}^{(\kappa)}\right)^q} \right)^{\theta} + \left(\frac{f\left(s_{\gamma 2}^{(\kappa)}\right)^q + f\left(s_{\delta 2}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma 2}^{(\kappa)}\right)^q - f\left(s_{\delta 2}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{1/\theta}} \right)^{\frac{1}{q}} \right) \left(\overline{p_{\delta 1}^{(\kappa)} + p_{\delta 2}^{(\kappa)}} \right) \right\} \right\rangle$$

$$\lambda \underset{D}{\odot} a = \left\langle \left\{ f^{-1}\left(\left(1-\frac{1}{1+\left(\lambda \left(\frac{f\left(s_{\gamma 1}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(p_{\gamma 1}^{(\kappa)} \right) \right\}, \left\{ f^{-1}\left(\left(\frac{1}{1+\left(\lambda \left(\frac{1-f\left(s_{\eta 1}^{(\kappa)}\right)^q}{f\left(s_{\eta 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(p_{\eta 1}^{(\kappa)} \right) \right\}, \right.$$

$$\left. \left\{ f^{-1}\left(\left(\frac{1}{1+\left(\lambda \left(\frac{f\left(s_{\gamma 1}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} - \frac{1}{1+\left(\lambda \left(\frac{f\left(s_{\delta 1}^{(\kappa)}\right)^q + f\left(s_{\gamma 1}^{(\kappa)}\right)^q}{1-f\left(s_{\delta 1}^{(\kappa)}\right)^q - f\left(s_{\gamma 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{1/\theta}} \right)^{\frac{1}{q}} \right) \left(p_{\delta 1}^{(\kappa)} \right) \right\} \right\rangle$$

$$a^{\lambda} = \left\langle \left\{ f^{-1}\left(\left(\frac{1}{1+\left(\lambda \left(\frac{1-f\left(s_{\gamma 1}^{(\kappa)}\right)^q}{f\left(s_{\gamma 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(p_{\gamma 1}^{(\kappa)} \right) \right\}, \left\{ f^{-1}\left(\left(1-\frac{1}{1+\left(\lambda \left(\frac{f\left(s_{\eta 1}^{(\kappa)}\right)^q}{1-f\left(s_{\eta 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(p_{\eta 1}^{(\kappa)} \right) \right\}, \right.$$

$$\left. \left\{ f^{-1}\left(\left(\frac{1}{1+\left(\lambda \left(\frac{f\left(s_{\eta 1}^{(\kappa)}\right)^q}{1-f\left(s_{\eta 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} - \frac{1}{1+\left(\lambda \left(\frac{f\left(s_{\delta 1}^{(\kappa)}\right)^q + f\left(s_{\eta 1}^{(\kappa)}\right)^q}{1-f\left(s_{\delta 1}^{(\kappa)}\right)^q - f\left(s_{\eta 1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(p_{\delta 1}^{(\kappa)} \right) \right\} \right\rangle$$

其中，$\widehat{p_{\gamma 1}+p_{\gamma 2}}=\left(p_{\gamma 1}^{(\kappa)}+p_{\gamma 2}^{(\kappa)}\right)\Big/\sum_{\kappa=1}^{L}\left(p_{\gamma 1}^{(\kappa)}+p_{\gamma 2}^{(\kappa)}\right)$。

定理 2 设 a,a_1,b_1 为论域 X 上的任意 3 个概率双层语言 T 球形模糊元，给定 $\lambda,\lambda_1,\lambda_2\ge 0$，则上述定义的运算满足以下性质：

1) $a\underset{D}{\oplus}b=b\underset{D}{\oplus}a$， $a\underset{D}{\otimes}b=b\underset{D}{\otimes}a$；

2) $\lambda\left(a\underset{D}{\oplus}b\right)=\lambda a\underset{D}{\oplus}\lambda b$， $\lambda_1 a\underset{D}{\oplus}\lambda_2 a=\left(\lambda_1+\lambda_2\right)\underset{D}{\odot}a$；

3) $\left(a\underset{D}{\otimes}b\right)^{\lambda}=a^{\lambda}\underset{D}{\otimes}b^{\lambda}$； $a^{\lambda_1}\underset{D}{\otimes}a^{\lambda_2}=a^{(\lambda_1+\lambda_2)}$。

4. 概率双层语言 T 球形模糊 Dombi 平均加权算子

定义 14 设 $a_i=\left\langle\left\{s_{\gamma i}^{(\kappa)}\left(p_{\gamma i}^{(\kappa)}\right)\right\},\left\{s_{\eta i}^{(\kappa)}\left(p_{\eta i}^{(\kappa)}\right)\right\},\left\{s_{\delta i}^{(\kappa)}\left(p_{\delta i}^{(\kappa)}\right)\right\}\right\rangle(i=1,2,\cdots,n)$ 为一组概率双层语言 T 球形模糊元，其对应的权重向量为 $w=\left(w_1,w_2,\cdots,w_n\right)^{\mathrm{T}}$，且满足 $w_i\ge 0,\sum_{i=1}^{n}w_i=1$，则概率双层语言 T 球形模糊 Dombi 加权平均算子定义为

$$PDHLTSF\text{-}DWA_w^{\theta}\left(a_1,a_2,\cdots,a_n\right)=\underset{D}{\oplus}_{i=1}^{n}w_i\underset{D}{\odot}a_i$$

定理 3 设 $a_i=\left\langle\left\{s_{\gamma i}^{(\kappa)}\left(p_{\gamma i}^{(\kappa)}\right)\right\},\left\{s_{\eta i}^{(\kappa)}\left(p_{\eta i}^{(\kappa)}\right)\right\},\left\{s_{\delta i}^{(\kappa)}\left(p_{\delta i}^{(\kappa)}\right)\right\}\right\rangle,i=1,2,\cdots,n$，则运用 $PDHLTSF\text{-}DWA_w^{\theta}$ 算子聚合后的结果仍是一个概率双层语言 T 球形模糊元，且

$$PDHLTF\text{-}DWA_w^{\theta}\left(a_1,a_2,\cdots,a_n\right)=\underset{D}{\oplus}_{i=1}^{n}w_i\underset{D}{\odot}a_i$$

$$=\left\langle\left\{f^{-1}\left(\left(1-\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma i}^{(\kappa)}\right)^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\widehat{\sum_{i=1}^{n}p_{\gamma i}^{(\kappa)}}\right)\right\},\right.$$

$$\left\{f^{-1}\left(\left(\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{1-f\left(s_{\eta i}^{(\kappa)}\right)^q}{f\left(s_{\eta i}^{(\kappa)}\right)^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\widehat{\sum_{i=1}^{n}p_{\eta i}^{(\kappa)}}\right)\right\},\tag{14}$$

$$\left.\left\{f^{-1}\left(\left(\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma i}^{(\kappa)}\right)^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}-\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{f\left(s_{\delta i}^{(\kappa)}\right)^q+f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\delta i}^{(\kappa)}\right)^q-f\left(s_{\gamma i}^{(\kappa)}\right)^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\widehat{\sum_{i=1}^{n}p_{\delta i}^{(\kappa)}}\right)\right\}\right\rangle$$

证明 当 $n=2$ 时，

$$
\begin{aligned}
& w_1 a_1 \oplus w_2 a_2 \\
& = \left\langle \left\{ f^{-1}\left(\left(1 - \frac{1}{1 + \left(\sum_{i=1}^{2} w_i \left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1 - f\left(s_{\gamma i}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(\overline{\sum_{i=1}^{2} p_{\gamma i}^{(\kappa)}} \right) \right\}, \right. \\
& \quad \left\{ f^{-1}\left(\left(\frac{1}{1 + \left(\sum_{i=1}^{2} w_i \left(\frac{1 - f\left(s_{\eta i}^{(\kappa)}\right)^q}{f\left(s_{\eta i}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(\overline{\sum_{i=1}^{2} p_{\eta i}^{(\kappa)}} \right) \right\}, \\
& \quad \left. \left\{ f^{-1}\left(\left(\frac{1}{1 + \left(\sum_{i=1}^{2} w_i \left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1 - f\left(s_{\gamma i}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} - \frac{1}{1 + \left(\sum_{i=1}^{2} w_i \left(\frac{f\left(s_{\delta i}^{(\kappa)}\right)^q + f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1 - f\left(s_{\delta i}^{(\kappa)}\right)^q - f\left(s_{\gamma i}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(\overline{\sum_{i=1}^{2} p_{\delta i}^{(\kappa)}} \right) \right\} \right\rangle
\end{aligned}
$$

即 $n=2$ 时，结论成立。假设 $n=k$ 时成立，下面证明当 $n=k+1$ 时，$PDHLTSF\text{-}DWA_w^{\theta}$ 算子仍然满足以上表达式。

$$
\begin{aligned}
& \left(\oplus_{i=1}^{k} w_i a_i \right) \oplus w_{k+1} a_{k+1} \\
& = \left\langle \left\{ f^{-1}\left(\left(1 - \frac{1}{1 + \left(\sum_{i=1}^{k} w_i \left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1 - f\left(s_{\gamma i}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(\overline{\sum_{i=1}^{k} p_{\gamma i}^{(\kappa)}} \right) \right\}, \left\{ f^{-1}\left(\left(\frac{1}{1 + \left(\sum_{i=1}^{k} w_i \left(\frac{1 - f\left(s_{\eta i}^{(\kappa)}\right)^q}{f\left(s_{\eta i}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(\overline{\sum_{i=1}^{k} p_{\eta i}^{(\kappa)}} \right) \right\}, \right. \\
& \quad \left. \left\{ f^{-1}\left(\left(\frac{1}{1 + \left(\sum_{i=1}^{k} w_i \left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1 - f\left(s_{\gamma i}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} - \frac{1}{1 + \left(\sum_{i=1}^{k} w_i \left(\frac{f\left(s_{\delta i}^{(\kappa)}\right)^q + f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1 - f\left(s_{\delta i}^{(\kappa)}\right)^q - f\left(s_{\gamma i}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(\overline{\sum_{i=1}^{k} p_{\delta i}^{(\kappa)}} \right) \right\} \right\rangle \\
& \oplus \left\langle \left\{ f^{-1}\left(\left(1 - \frac{1}{1 + \left(w_{k+1} \left(\frac{f\left(s_{\gamma k+1}^{(\kappa)}\right)^q}{1 - f\left(s_{\gamma k+1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(p_{\gamma k+1}^{(\kappa)} \right) \right\}, \left\{ f^{-1}\left(\left(\frac{1}{1 + \left(w_{k+1} \left(\frac{1 - f\left(s_{\eta k+1}^{(\kappa)}\right)^q}{f\left(s_{\eta k+1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(p_{\eta k+1}^{(\kappa)} \right) \right\}, \right. \\
& \quad \left. \left\{ f^{-1}\left(\left(\frac{1}{1 + \left(w_{k+1} \left(\frac{f\left(s_{\gamma k+1}^{(\kappa)}\right)^q}{1 - f\left(s_{\gamma k+1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} - \frac{1}{1 + \left(w_{k+1} \left(\frac{f\left(s_{\delta k+1}^{(\kappa)}\right)^q + f\left(s_{\gamma k+1}^{(\kappa)}\right)^q}{1 - f\left(s_{\delta k+1}^{(\kappa)}\right)^q - f\left(s_{\gamma k+1}^{(\kappa)}\right)^q} \right)^{\theta} \right)^{\frac{1}{\theta}}} \right)^{\frac{1}{q}} \right) \left(p_{\delta k+1}^{(\kappa)} \right) \right\} \right\rangle
\end{aligned}
$$

$$=PDHLTSF-DWA_w^{\theta}\left(a_1,a_2,\cdots,a_{k+1}\right)$$

因此当 n 为任意正整数时，式(14)成立。下证 $PDHLTF\text{-}DWA_w^{\theta}$ 算子聚合后的结果仍是概率双层语言 T 球形模糊元。

令 $PDHLTF\text{-}DWA_w^{\theta}\left(a_2,\cdots,a_n\right)=\oplus_{i=1}^{n}w_i\odot a_i=\left\langle \widetilde{H_s},\widetilde{G_s},\widetilde{I_s}\right\rangle$，若要证明通过 $PDHLTF\text{-}DWA_w^{\theta}$ 算子聚合后的结果仍是一个概率双层语言 T 球形模糊元，则需证明 $\widetilde{H_s},\widetilde{G_s},\widetilde{I_s}$ 满足：

1) $\sum\widetilde{p_{\gamma}}\le 1,\sum\widetilde{p_{\eta}}\le 1,\sum\widetilde{p_{\delta}}\le 1$；

2) $0\le\left(\tilde{\alpha}^{+}\right)^q+\left(\tilde{\beta}^{+}\right)^q+\left(\tilde{\chi}^{+}\right)^q\le 1$，其中 $\tilde{\alpha}^{+}=\max_{\kappa}f\left(\widetilde{s_{\gamma}^{(\kappa)}}\right),\tilde{\beta}^{+}=\max_{\kappa}f\left(\widetilde{s_{\eta}^{(\kappa)}}\right),\tilde{\chi}^{+}=\max_{\kappa}f\left(\widetilde{s_{\delta}^{(\kappa)}}\right)$。

由定义 14 可知 $a_i=\left\langle H_{si},G_{si},I_{si}\right\rangle=\left\langle\left\{s_{\gamma i}^{(\kappa)}\left(p_{\gamma i}^{(\kappa)}\right)\right\},\left\{s_{\eta i}^{(\kappa)}\left(p_{\eta i}^{(\kappa)}\right)\right\},\left\{s_{\delta i}^{(\kappa)}\left(p_{\delta i}^{(\kappa)}\right)\right\}\right\rangle$，$i=1,\cdots,n$，因此 $s_{\gamma i}^{(\kappa)}\subseteq S_O$，$s_{\eta i}^{(\kappa)}\subseteq S_O$，$s_{\delta i}^{(\kappa)}\subseteq S_O$，故 $f\left(s_{\gamma i}^{(\kappa)}\right),f\left(s_{\eta i}^{(\kappa)}\right),f\left(s_{\delta k}^{(\kappa)}\right)\in[0,1]$。当 $q\ge 1$ 时，$f\left(s_{\gamma i}^{(\kappa)}\right)^q,f\left(s_{\eta i}^{(\kappa)}\right)^q,f\left(s_{\delta k}^{(\kappa)}\right)^q$ 的取值范围仍在 0 到 1 之间。

现考虑一个多元函数的单调性和值域，

$$g\left(x_1,x_1,\cdots,x_n\right)=1-\left(1+\left(\sum_{i=1}^{n}c_i\left(\frac{x_i}{1-x_i}\right)^{\theta}\right)^{\frac{1}{\theta}}\right)^{-1}$$

其中，$c_i\ge 0$，且 $\sum_{i=1}^{n}c_i=1,\theta\ge 1,x_i\in[0,1]$。易知该函数关于 x_i 是单调递增函数。则该函数在 $\left(x_1^{+},x_2^{+},\cdots,x_n^{+}\right)$ 点取得最大值，其中 $x_i^{+}=\sup x_i,i=1,\cdots,n$。令 $h(x)=1-g\left(x^{-1}\right)$，由复合函数的单调性，可得多元函数 h 也在区间上单调递增。

令 $\tilde{\alpha}^{+}=\max_{\kappa}f\left(\widetilde{s_{\gamma}^{(\kappa)}}\right),\tilde{\beta}^{+}=\max_{\kappa}f\left(\widetilde{s_{\eta}^{(\kappa)}}\right),\tilde{\chi}^{+}=\max_{\kappa}f\left(\widetilde{s_{\delta}^{(\kappa)}}\right)$，对于任意一个 $\widetilde{s_{\gamma}^{(\kappa)}},\kappa=1,2,\cdots,L$，

$$f\left(\widetilde{s_{\gamma}^{(\kappa)}}\right)=f\left(f^{-1}\left(\left(1-\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma}^{(\kappa)}\right)^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\right)=\left(1-\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma}^{(\kappa)}\right)^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}$$

从上文的多元函数单调性讨论，发现

$$\tilde{\alpha}^{+}=\left(1-\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{\overline{f\left(s_{\gamma i}\right)}^q}{1-\overline{f\left(s_{\gamma i}\right)}^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}},\tilde{\beta}^{+}=\left(\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{1-\overline{f\left(s_{\eta i}\right)}^q}{\overline{f\left(s_{\eta i}\right)}^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}$$

$$\tilde{\chi}^{+}=\left(\frac{1}{1+\left(\sum_{i=1}^{k}w_i\left(\frac{\overline{f\left(s_{\gamma i}\right)}^q}{1-\overline{f\left(s_{\gamma i}\right)}^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}-\frac{1}{1+\left(\sum_{i=1}^{k}w_i\left(\frac{\overline{f\left(s_{\delta i}\right)}^q+\overline{f\left(s_{\gamma i}\right)}^q}{1-\overline{f\left(s_{\delta i}\right)}^q-\overline{f\left(s_{\gamma i}\right)}^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}$$

其中 $\overline{f\left(s_{\gamma i}\right)}=\max_{\kappa}f\left(s_{\gamma i}^{(\kappa)}\right),\overline{f\left(s_{\eta i}\right)}=\max_{\kappa}f\left(s_{\eta i}^{(\kappa)}\right),\overline{f\left(s_{\delta i}\right)}=\max_{\kappa}f\left(s_{\delta i}^{(\kappa)}\right),i=1,\cdots,n$。

由概率双层语言 T 球形模糊集的定义，可知对所有的 $i(i=1,2,\cdots,n)$ 有 $\overline{f(s_{\gamma i})}^q+\overline{f(s_{\eta i})}^q+\overline{f(s_{\delta i})}^q\le 1$ 成立，进而

$$\overline{f(s_{\gamma i})}^q\le 1-\overline{f(s_{\eta i})}^q-\overline{f(s_{\delta i})}^q\Rightarrow\left(\frac{1-\overline{f(s_{\eta i})}^q}{\overline{f(s_{\eta i})}^q}\right)^\theta\ge\left(\frac{\overline{f(s_{\gamma i})}^q+\overline{f(s_{\delta i})}^q}{1-\overline{f(s_{\delta i})}^q-\overline{f(s_{\gamma i})}^q}\right)^\theta$$

$$\Rightarrow\frac{1}{1+\left(\sum_{i=1}^n w_i\left(\frac{1-\overline{f(s_{\eta i})}^q}{\overline{f(s_{\eta i})}^q}\right)^\theta\right)^{\frac{1}{\theta}}}-\frac{1}{1+\left(\sum_{i=1}^k w_i\left(\frac{\overline{f(s_{\delta i})}^q+\overline{f(s_{\gamma i})}^q}{1-\overline{f(s_{\delta i})}^q-\overline{f(s_{\gamma i})}^q}\right)^\theta\right)^{\frac{1}{\theta}}}\le 0$$

然后

$$\left(\tilde{\alpha}^+\right)^q+\left(\tilde{\beta}^+\right)^q+\left(\tilde{\chi}^+\right)^q=1+\frac{1}{1+\left(\sum_{i=1}^n w_i\left(\frac{1-\overline{f(s_{\eta i})}^q}{\overline{f(s_{\eta i})}^q}\right)^\theta\right)^{\frac{1}{\theta}}}-\frac{1}{1+\left(\sum_{i=1}^k w_i\left(\frac{\overline{f(s_{\delta i})}^q+\overline{f(s_{\gamma i})}^q}{1-\overline{f(s_{\delta i})}^q-\overline{f(s_{\gamma i})}^q}\right)^\theta\right)^{\frac{1}{\theta}}}$$

因此得到 $\left(\tilde{\alpha}^+\right)^q+\left(\tilde{\beta}^+\right)^q+\left(\tilde{\chi}^+\right)^q\le 1$。

又因为 $\overline{f(s_{\gamma i})}^q,\overline{f(s_{\eta i})}^q,\overline{f(s_{\delta i})}^q\ge 0$， $w_i\ge 0$，显然 $\left(\tilde{\alpha}^+\right)^q+\left(\tilde{\beta}^+\right)^q+\left(\tilde{\chi}^+\right)^q\ge 0$，故定理 3 证明完成。

性质 1 (幂等性) 设 $a=\left\langle\left\{s_\gamma^{(\kappa)}\left(p_\gamma^{(\kappa)}\right)\right\},\left\{s_\eta^{(\kappa)}\left(p_\eta^{(\kappa)}\right)\right\},\left\{s_\delta^{(\kappa)}\left(p_\delta^{(\kappa)}\right)\right\}\right\rangle$ 是一个概率双层语言 T 球形模糊元，$a_i=\left\langle\left\{s_{\gamma i}^{(\kappa)}\left(p_{\gamma i}^{(\kappa)}\right)\right\},\left\{s_{\eta i}^{(\kappa)}\left(p_{\eta i}^{(\kappa)}\right)\right\},\left\{s_{\delta i}^{(\kappa)}\left(p_{\delta i}^{(\kappa)}\right)\right\}\right\rangle(i=1,2,\cdots,n)$ 为一组概率双层语言 T 球形模糊元，若 $a_i=a(i=1,2,\cdots,n)$，则

$$PDHLTSF\text{-}DWA_w^\theta\left(a_1,a_2,\cdots,a_n\right)=a$$

证明 由条件可得，对所有的 $i(i=1,2,\cdots,n)$，有 $s_{\gamma i}^{(\kappa)}=s_\gamma^{(\kappa)},s_{\eta i}^{(\kappa)}=s_\eta^{(\kappa)},s_{\delta i}^{(\kappa)}=s_\delta^{(\kappa)},p_{\gamma i}^{(\kappa)}=p_\gamma^{(\kappa)},p_{\eta i}^{(\kappa)}=p_\eta^{(\kappa)},p_{\delta i}^{(\kappa)}=p_\delta^{(\kappa)}$ 成立。则

$$\begin{aligned}&PDHLTSF\text{-}DWA_w^\theta\left(a_1,a_2,\cdots,a_n\right)\\&=\left\langle\left\{f^{-1}\left(\left(1-\frac{1}{1+\left(\sum_{i=1}^n w_i\left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma i}^{(\kappa)}\right)^q}\right)^\theta\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\overline{\sum_{i=1}^n p_{\gamma i}^{(\kappa)}}\right)\right\},\right.\\&\left\{f^{-1}\left(\left(\frac{1}{1+\left(\sum_{i=1}^n w_i\left(\frac{1-f\left(s_{\eta i}^{(\kappa)}\right)^q}{f\left(s_{\eta i}^{(\kappa)}\right)^q}\right)^\theta\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\overline{\sum_{i=1}^n p_{\eta i}^{(\kappa)}}\right)\right\},\\&\left.\left\{f^{-1}\left(\left(\frac{1}{1+\left(\sum_{i=1}^n w_i\left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma i}^{(\kappa)}\right)^q}\right)^\theta\right)^{\frac{1}{\theta}}}-\frac{1}{1+\left(\sum_{i=1}^n w_i\left(\frac{f\left(s_{\delta i}^{(\kappa)}\right)^q+f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\delta i}^{(\kappa)}\right)^q-f\left(s_{\gamma i}^{(\kappa)}\right)^q}\right)^\theta\right)^{\frac{1}{\theta}}}\right)^{\frac{1}{q}}\right)\left(\overline{\sum_{i=1}^n p_{\delta i}^{(\kappa)}}\right)\right\}\right\rangle\\&=\left\langle\left\{s_\gamma^{(\kappa)}\left(p_\gamma^{(\kappa)}\right)\right\},\left\{s_\eta^{(\kappa)}\left(p_\eta^{(\kappa)}\right)\right\},\left\{s_\delta^{(\kappa)}\left(p_\delta^{(\kappa)}\right)\right\}\right\rangle=a\end{aligned}$$

性质 2 (单调性)设 $a_i=\left\langle\left\{s_{\gamma i}^{(\kappa)}\left(p_{\gamma}^{(\kappa)}\right)\right\},\left\{s_{\eta i}^{(\kappa)}\left(p_{\eta i}^{(\kappa)}\right)\right\},\left\{s_{\delta i}^{(\kappa)}\left(p_{\delta i}^{(\kappa)}\right)\right\}\right\rangle$ 和 $b_i=\left\langle\left\{\tilde{s}_{\gamma i}^{(\kappa)}\left(\tilde{p}_{\gamma i}^{(\kappa)}\right)\right\},\left\{\tilde{s}_{\eta i}^{(\kappa)}\left(\tilde{p}_{\eta i}^{(\kappa)}\right)\right\},\left\{\tilde{s}_{\delta i}^{(\kappa)}\left(\tilde{p}_{\delta i}^{(\kappa)}\right)\right\}\right\rangle$ $(i=1,2,\cdots,n)$ 为两组概率双层语言 T 球形模糊元，若对于任意的 $i\,(i=1,2,\cdots,n)$，都有 $s_{\gamma i}^{(\kappa)}\le\tilde{s}_{\gamma i}^{(\kappa)},s_{\eta i}^{(\kappa)}\ge\tilde{s}_{\eta i}^{(\kappa)},s_{\delta i}^{(\kappa)}\ge\tilde{s}_{\delta i}^{(\kappa)}$ 成立，对应的概率同样满足 $p_{\gamma i}^{(\kappa)}\le\tilde{p}_{\gamma i}^{(\kappa)},p_{\eta i}^{(\kappa)}\ge\tilde{p}_{\eta i}^{(\kappa)},p_{\delta i}^{(\kappa)}\ge\tilde{p}_{\delta i}^{(\kappa)}$，则

$$PDHLTSF\text{-}DWA_w^{\theta}\left(a_1,a_2,\cdots,a_n\right)\le PDHLTF\text{-}DWA_w^{\theta}\left(b_1,b_2,\cdots,b_n\right)$$

证明 设 $PDHLTF\text{-}DWA_w^{\theta}\left(a_1,a_2,\cdots,a_n\right),PDHLTF\text{-}DWA_w^{\theta}\left(b_1,b_2,\cdots,b_n\right)$ 聚合后的结果为 $a=\left\langle\left\{\widehat{s_{\gamma i}^{(\kappa)}}\right\},\left\{\widehat{s_{\eta i}^{(\kappa)}}\right\},\left\{\widehat{s_{\delta i}^{(\kappa)}}\right\}\right\rangle,b=\left\langle\left\{\widehat{\tilde{s}_{\gamma i}^{(\kappa)}}\right\},\left\{\widehat{\tilde{s}_{\eta i}^{(\kappa)}}\right\},\left\{\widehat{\tilde{s}_{\delta i}^{(\kappa)}}\right\}\right\rangle$。对于性质 2 给定条件，可以推出：$f\left(s_{\gamma i}^{(\kappa)}\right)\le f\left(\tilde{s}_{\gamma i}^{(\kappa)}\right)$，$f\left(s_{\eta i}^{(\kappa)}\right)\ge f\left(\tilde{s}_{\eta i}^{(\kappa)}\right),f\left(s_{\delta i}^{(\kappa)}\right)\ge f\left(\tilde{s}_{\delta i}^{(\kappa)}\right),i=1,2,\cdots,n$。

由定理 3 证明多元函数的单调性讨论可得

$$\sqrt[q]{1-\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{f\left(s_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(s_{\gamma i}^{(\kappa)}\right)^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}}\le\sqrt[q]{1-\frac{1}{1+\left(\sum_{i=1}^{n}w_i\left(\frac{f\left(\tilde{s}_{\gamma i}^{(\kappa)}\right)^q}{1-f\left(\tilde{s}_{\gamma i}^{(\kappa)}\right)^q}\right)^{\theta}\right)^{\frac{1}{\theta}}}}$$

同时聚合后的概率满足 $\widehat{\sum_{i=1}^{n}p_{\gamma i}^{(\kappa)}}\le\widehat{\sum_{i=1}^{n}\tilde{p}_{\gamma i}^{(\kappa)}}$，同理对于 $s_{\eta i}^{(\kappa)},\tilde{s}_{\eta i}^{(\kappa)}$ 以及 $s_{\delta i}^{(\kappa)},\tilde{s}_{\delta i}^{(\kappa)}$，经过 $PDHLTF\text{-}DWA_w^{\theta}$ 聚合后，仍满足上述不等式。最后由式(11)计算可得性质 2 成立。

性质 3 (有界性)设 $a_i=\left\langle\left\{s_{\gamma i}^{(\kappa)}\left(p_{\gamma}^{(\kappa)}\right)\right\},\left\{s_{\eta i}^{(\kappa)}\left(p_{\eta}^{(\kappa)}\right)\right\},\left\{s_{\delta i}^{(\kappa)}\left(p_{\delta}^{(\kappa)}\right)\right\}\right\rangle$ 是一组模糊元，令

$a^{+}=\left\langle\left\{\max\limits_{\kappa}s_{\gamma i}^{(\kappa)}\max\limits_{\kappa}\left(p_{\gamma i}^{(\kappa)}\right)\right\},\left\{\min\limits_{\kappa}s_{\eta i}^{(\kappa)}\min\limits_{\kappa}\left(p_{\eta i}^{(\kappa)}\right)\right\},\left\{\min\limits_{\kappa}s_{\delta i}^{(\kappa)}\min\limits_{\kappa}\left(p_{\delta i}^{(\kappa)}\right)\right\}\right\rangle$，

$a^{-}=\left\langle\left\{\min\limits_{\kappa}s_{\gamma i}^{(\kappa)}\min\limits_{\kappa}\left(p_{\gamma i}^{(\kappa)}\right)\right\},\left\{\max\limits_{\kappa}s_{\eta i}^{(\kappa)}\max\limits_{\kappa}\left(p_{\eta i}^{(\kappa)}\right)\right\},\left\{\max\limits_{\kappa}s_{\delta i}^{(\kappa)}\max\limits_{\kappa}\left(p_{\delta i}^{(\kappa)}\right)\right\}\right\rangle$，则

$$a^{-}\le PDHLTSF\text{-}DWA_w^{\theta}\left(a_1,a_2,\cdots,a_n\right)\le a^{+}$$

证明 由性质 1 可知 $a^{-}=PDHLTF\text{-}DWA_w^{\theta}\left(\underbrace{a^{-},a^{-},\cdots,a^{-}}_{n}\right)$，$a^{+}$ 同理。然后利用性质 2 的单调性，易证有界性成立。

5. 基于 PDHLTF-DWA 算子的多属性决策方法

5.1. 决策者权重确定

定义 15 [18]设 $e=\{e_1,\cdots,e_n\}$ 为一决策者集合，决策者 h 对决策者 k 的信任度定义为

$$TD_{hk}=CI\left(e_{hk}\right)\cdot R_{hk}\tag{15}$$

其中，R_{hk} 为社交网络中决策者 h 与决策者 k 的关系强度，$CI\left(e_{hk}\right)$ 为一致性度量，计算公式如下：

$$CI\left(e_{hk}\right)=1-\frac{1}{m\times n}\sum_{i=1}^{m}\sum_{j=1}^{n}d\left(a_{ij}^{h},a_{ij}^{k}\right)\tag{16}$$

则得到关于决策者的信任度矩阵

$$TD=\begin{pmatrix}TD_{11}&\cdots&TD_{1n}\\\vdots&\ddots&\vdots\\TD_{m1}&\cdots&TD_{mn}\end{pmatrix}$$

计算各决策者的总信任度

$$TS_i = \frac{1}{n}\sum_{j=1}^{n} TD_{ij} \tag{17}$$

然后得到决策者主观权重

$$W_{1i} = \frac{TS_i}{\sum_{i=1}^{n} TS_i} \tag{18}$$

定义 16 [19]设 u 为特征函数，决策者 i 的 Shapley 值定义为

$$w_{si}(u, X) = \sum_{S \subseteq X \setminus i} \frac{(n-s-1)!s!}{n!}\left(u(S \cup i) - u(S)\right) \tag{19}$$

其中 X 是所有决策者集合，n, s 分别为集合 X, S 的元素个数。

决策者的综合权重计算公式如下

$$W_{ei} = \sigma W_{1i} + (1-\sigma) W_{2i} \tag{20}$$

其中，$\sigma \in [0,1]$ 为组合权重偏好程度，W_{2i} 为标准化后的 w_{si}。

5.2. 属性权重确定

在处理多属性决策问题时，各个属性的权重对最终结果具有显著影响。因此，确定合适的权重对于提高决策效率至关重要。客观权重法能够基于数据本身来确定权重，从而更准确地反映数据的关键特征，这种方法因其客观性而被广泛采用。熵权法通过信息熵的概念来评估属性的价值，而 CILOS 法则是从损失的角度来衡量属性的重要性，这两种方法各有其优势和特点。IDOCRIW [20]作为一种结合熵权法和 CILOS 的权重确定方法，融合了两种方法的优点。

具体步骤如下：

步骤 1：对决策矩阵 $X = (x_{ij})_{m \times n}$，首先标准化 X，得到 $Y = (y_{ij})_{m \times n}$。针对效益性属性 B 和成本性属性 C，其标准化公式如下：

$$y_{ij} = \begin{cases} \dfrac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}}, j \in B, \\ \dfrac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}}, j \in C. \end{cases} \tag{21}$$

步骤 2：计算第 j 个属性下的熵值

$$e_j = -\frac{1}{\ln m}\sum_{i=1}^{m} y_{ij} \ln y_{ij} \tag{22}$$

步骤 3：根据熵权法得到各属性的权重为

$$w_j = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)} \tag{23}$$

步骤 4：计算每个属性下的最优值 $r_j = \max_i y_{ij} = r_{k_j j}$，其中 k_j 为第 j 个属性最优值所在的行。并构造方阵 $B = (b_{ij})_{n \times n}, b_{ij} = r_{k_i j}, b_{ii} = r_i$，$r_{k_i j}$ 表示第 i 个属性最优值所在行和第 j 个属性所在列的值。

步骤 5：构造属性之间的相对损失矩阵

$$Z=\left(z_{ij}\right)_{n\times n}, z_{ij}=\frac{r_j-b_{ij}}{r_j}=\frac{b_{ii}-b_{ij}}{b_{ii}} \tag{24}$$

其中，z_{ij} 表示当第 i 个属性处于最优值时，第 j 个属性的显著损失。

步骤 6：构建一个齐次线性方程组，求解属性损失权重

$$F=\left(f_{ij}\right)_{n\times n}\text{，当 } i=j \text{ 时，} f_{ij}=-\sum_{i=1}^{n} z_{ij}\text{；当 } i\neq j, f_{ij}=z_{ij}\text{。}$$

设权重向量为 $q=\left(q_1,q_2,\cdots,q_n\right)^{\mathrm{T}}$，求解 $F\cdot q=0$，则标准化后第 j 个属性的损失权重为

$$\omega_j=\frac{q_j}{\sum_{j=1}^{n} q_j} \tag{25}$$

最后，综合熵权法权重和损失权重，属性的最终权重为

$$W_j=\frac{w_j\omega_j}{\sum_{j=1}^{n} w_j\omega_j} \tag{26}$$

5.3. 基于 PDHLTF-DWA 的决策步骤

假设备选方案集合为 $A=\left\{A_1,A_2,\cdots,A_n\right\}$，属性集合为 $C=\left\{C_1,C_2,\cdots,C_m\right\}$，其中有 l 个决策者 $E=\left\{E_1,E_2,\cdots,E_l\right\}$，第 k 个决策者给出的评价信息矩阵为 $P^k=\left(a_{ij}^k\right)_{m\times n}$。

决策流程如图 1 所示，具体的决策步骤如下：

步骤 1：决策者根据给出的语言尺度，对具体问题进行评价，得到决策矩阵。首先对决策矩阵进行标准化处理，然后对概率双层语言 T 球形模糊元规范化。

步骤 2：根据 5.1 节的方法，计算决策者的信任度和 Shapley 值，得到决策者综合权重。

步骤 3：利用 $PDHLTF\text{-}DWA_w^{\theta}$ 算子，将由信任度得到的决策者权重进行聚合，得到综合决策矩阵。

步骤 4：利用 IDOCRIW 方法得到各属性的权重后，使用 $PDHLTF\text{-}DWA_w^{\theta}$ 算子聚合各属性，得到综合决策 PDHLTSFE。

步骤 5：最后利用式(11)计算各备选方案的综合评价值，根据排序规则，得到最优方案。

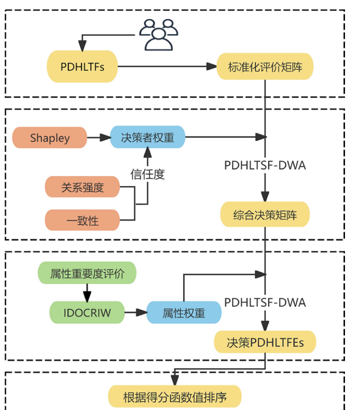


Figure 1. Decision-making process
图 1. 决策步骤

6. 实例应用

6.1. 数值算例

近几年，随着网络新兴媒介快速发展以及各地文旅部门的积极宣传，催生出众多网红城市和景点。随之也产生了大量旅游安全问题，这不仅对游客的生命安全产生威胁，也对旅游业的可持续发展产生重大影响。在一些景区发生的安全事故给游客的人身和财产带来了极大的威胁，若能提前对景区的安全风险进行评估，既能提高游客的信任度，也能使景区管理防患于未然，提升景区安全系数。因此对于景区的安全评估是十分有必要的。

基于以往的景区事故分析[21]，景区的安全评估指标主要包括 5 个因素，分别为景点安全管理(C_1)、设施设备(C_2)、游客行为(C_3)、环境安全(C_4)、安全监控和预警(C_5)。

设有某 3 个待评估的网红景点 A_1, A_2, A_3，关于评价信息的语言尺度定义如下，第一层语言术语集 $S=\{s_{-2}=\text{"很差"}, s_{-1}=\text{"差"}, s_0=\text{"中等"}, s_1=\text{"一般"}, s_2=\text{"好"}\}$，第二层语言术语集 $O=\{o_{-2}=\text{"远不是"}, o_{-1}=\text{"有点"}, o_0=\text{"正好"}, o_1=\text{"非常"}, o_2=\text{"完全"}\}$。现邀请专家 $E_i\,(i=1,2,3)$ 考虑以上 5 个因素对该 3 个景点进行评价，专家以概率双层 T 球形模糊语言集的形式给出其评价信息。

令 $0<x\le y\le 1$，可得 $TN(x,y)$ 和 $TCN(x,y)$ 的取值范围，$0\le TN(x,y)\le\left(1+2^{\frac{1}{r}}\left(\frac{1-y}{y}\right)\right)^{-1}$，$\left(1+2^{\frac{1}{r}}\left(\frac{x}{1-x}\right)\right)^{-1}\le TCN(x,y)\le 1$。当 Dombi 范数中的参数 $\theta\to 0^+$ 时，$TN(x,y)\to 0$，$TCN(x,y)\to 1$，因此，θ 值越小，Dombi 范数趋近于最小值函数，在决策过程中意味着这类决策者更注重避免损失，属于风险规避型决策者。而当 $\theta\to+\infty$ 时，$TN(x,y)\to 1$，$TCN(x,y)\to 0$，Dombi 范数趋近于最大值函数，意味着这类决策者愿意承担较高的风险，属于风险偏好型决策者。在实际决策中，可根据具体的决策情境和风险承受能力来合理选择参数 θ。6.2 节给出 θ 变化对决策结果的影响。

在这个案例中，考虑决策者以相对保守的态度参与决策过程，参数值取 $q=2, \theta=1, \sigma=0.5$。按照 5.3 的决策步骤：

步骤 1：将获取的概率双层语言 T 球形模糊评价信息标准化和规范化，其中安全评估的所有属性均为效益型属性，见表 1~3。

Table 1. Standardized evaluation information of experts E_1

表 1. 专家 E_1 的标准化后的评价信息

	C_1	C_2	C_3	C_4	C_5
A_1	$\begin{bmatrix}\{s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0.2), s_{-1\langle o_2\rangle}(0.8)\},\\ \{s_{-1\langle o_1\rangle}(0), s_{-1\langle o_1\rangle}(0.35), s_{1\langle o_0\rangle}(0.65)\},\\ \{s_{-1\langle o_0\rangle}(0), s_{-1\langle o_0\rangle}(0), s_{-1\langle o_0\rangle}(0.65)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{0\langle o_2\rangle}(0), s_{1\langle o_1\rangle}(0.24), s_{0\langle o_2\rangle}(0.68)\},\\ \{s_{2\langle o_{-2}\rangle}(0), s_{2\langle o_{-2}\rangle}(0), s_{2\langle o_{-2}\rangle}(0.6)\},\\ \{s_{-1\langle o_1\rangle}(0), s_{-1\langle o_1\rangle}(0.3), s_{0\langle o_{-2}\rangle}(0.45)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0.7), s_{1\langle o_0\rangle}(0.3)\},\\ \{s_{-2\langle o_0\rangle}(0), s_{-2\langle o_0\rangle}(0.2), s_{-1\langle o_{-2}\rangle}(0.75)\},\\ \{s_{0\langle o_{-2}\rangle}(0), s_{0\langle o_{-2}\rangle}(0.45), s_{1\langle o_{-2}\rangle}(0.55)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{1\langle o_{-1}\rangle}(0), s_{1\langle o_{-1}\rangle}(0), s_{1\langle o_{-1}\rangle}(1)\},\\ \{s_{-1\langle o_0\rangle}(0), s_{-1\langle o_0\rangle}(0.56), s_{0\langle o_{-2}\rangle}(0.44)\},\\ \{s_{1\langle o_{-2}\rangle}(0), s_{1\langle o_{-2}\rangle}(0), s_{1\langle o_{-2}\rangle}(0.65)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{1\langle o_{-2}\rangle}(0.1), s_{0\langle o_{-1}\rangle}(0.3), s_{2\langle o_{-1}\rangle}(0.6)\},\\ \{s_{0\langle o_{-2}\rangle}(0), s_{0\langle o_{-2}\rangle}(0), s_{0\langle o_{-2}\rangle}(1)\},\\ \{s_{-2\langle o_2\rangle}(0), s_{-2\langle o_2\rangle}(0), s_{-2\langle o_2\rangle}(0.7)\}\end{bmatrix}$
A_2	$\begin{bmatrix}\{s_{0\langle o_{-2}\rangle}(0.2), s_{1\langle o_{-2}\rangle}(0.4), s_{0\langle o_1\rangle}(0.4)\},\\ \{s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0.5), s_{1\langle o_{-2}\rangle}(0.5)\},\\ \{s_{0\langle o_{-1}\rangle}(0), s_{0\langle o_{-1}\rangle}(0), s_{0\langle o_{-1}\rangle}(0.6)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0.1), s_{1\langle o_{-1}\rangle}(0.9)\},\\ \{s_{-1\langle o_0\rangle}(0.2), s_{2\langle o_{-1}\rangle}(0.2), s_{1\langle o_{-1}\rangle}(0.4)\},\\ \{s_{-2\langle o_0\rangle}(0), s_{-2\langle o_0\rangle}(0.6), s_{1\langle o_0\rangle}(0.4)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{-1\langle o_0\rangle}(0), s_{-1\langle o_0\rangle}(0.7), s_{1\langle o_1\rangle}(0.3)\},\\ \{s_{1\langle o_{-1}\rangle}(0.2), s_{2\langle o_{-1}\rangle}(0.2), s_{-1\langle o_2\rangle}(0.6)\},\\ \{s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0.6)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{0\langle o_1\rangle}(0), s_{0\langle o_1\rangle}(0.4), s_{1\langle o_{-2}\rangle}(0.6)\},\\ \{s_{0\langle o_{-1}\rangle}(0), s_{0\langle o_{-1}\rangle}(0.2), s_{0\langle o_1\rangle}(0.8)\},\\ \{s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0.3), s_{0\langle o_{-2}\rangle}(0.7)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{1\langle o_1\rangle}(0), s_{1\langle o_1\rangle}(0.2), s_{2\langle o_{-1}\rangle}(0.8)\},\\ \{s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0.8), s_{1\langle o_0\rangle}(0.2)\},\\ \{s_{-2\langle o_0\rangle}(0), s_{-2\langle o_0\rangle}(0.4), s_{-2\langle o_1\rangle}(0.6)\}\end{bmatrix}$
A_3	$\begin{bmatrix}\{s_{0\langle o_{-2}\rangle}(0), s_{0\langle o_{-2}\rangle}(0), s_{0\langle o_{-2}\rangle}(0.5)\},\\ \{s_{-2\langle o_0\rangle}(0), s_{-2\langle o_0\rangle}(0.2), s_{-1\langle o_{-1}\rangle}(0.8)\},\\ \{s_{-2\langle o_2\rangle}(0), s_{-2\langle o_2\rangle}(0.6), s_{-1\langle o_1\rangle}(0.4)\}\end{bmatrix}$	$\begin{Bmatrix}\{s_{-1\langle o_0\rangle}(0), s_{-1\langle o_0\rangle}(0), s_{-1\langle o_0\rangle}(0.3)\},\\ \{s_{-2\langle o_0\rangle}(0), s_{-2\langle o_0\rangle}(0), s_{-2\langle o_0\rangle}(0.7)\},\\ \{s_{1\langle o_{-1}\rangle}(0), s_{1\langle o_{-1}\rangle}(0), s_{1\langle o_{-1}\rangle}(0.65)\}\end{Bmatrix}$	$\begin{bmatrix}\{s_{2\langle o_{-1}\rangle}(0), s_{2\langle o_{-1}\rangle}(0.3), s_{1\langle o_{-1}\rangle}(0.4)\},\\ \{s_{-1\langle o_0\rangle}(0), s_{-1\langle o_0\rangle}(0.15), s_{0\langle o_1\rangle}(0.85)\},\\ \{s_{-2\langle o_1\rangle}(0), s_{-2\langle o_1\rangle}(0.5), s_{-1\langle o_{-1}\rangle}(0.5)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{2\langle o_{-2}\rangle}(0), s_{2\langle o_{-2}\rangle}(0.75), s_{1\langle o_1\rangle}(0.1)\},\\ \{s_{1\langle o_0\rangle}(0), s_{1\langle o_0\rangle}(0.5), s_{2\langle o_{-1}\rangle}(0.5)\},\\ \{s_{-2\langle o_0\rangle}(0), s_{-2\langle o_0\rangle}(0), s_{-2\langle o_0\rangle}(0.7)\}\end{bmatrix}$	$\begin{bmatrix}\{s_{-2\langle o_1\rangle}(0), s_{-2\langle o_1\rangle}(0.3), s_{2\langle o_{-1}\rangle}(0.3)\},\\ \{s_{-1\langle o_{-1}\rangle}(0), s_{-1\langle o_{-1}\rangle}(0.4), s_{1\langle o_{-2}\rangle}(0.6)\},\\ \{s_{-2\langle o_0\rangle}(0.3), s_{-2\langle o_1\rangle}(0.1), s_{-1\langle o_1\rangle}(0.6)\}\end{bmatrix}$

Table 2. Standardized evaluation information of experts E_2

表 2. 专家 E_2 的标准化后的评价信息

	C_1	C_2	C_3	C_4	C_5
A_1	$\begin{bmatrix} \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.2),s_{-1\langle o_2\rangle}(0.8)\}, \\ \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.25),s_{1\langle o_{-2}\rangle}(0.65)\}, \\ \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.7)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.4),s_{1\langle o_{-2}\rangle}(0.6)\}, \\ \{s_{1\langle o_{-2}\rangle}(0),s_{1\langle o_{-2}\rangle}(0),s_{1\langle o_{-2}\rangle}(0.6)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0),s_{-1\langle o_{-2}\rangle}(0.4),s_{0\langle o_{-2}\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.7),s_{0\langle o_1\rangle}(0.3)\}, \\ \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.2),s_{1\langle o_{-1}\rangle}(0.65)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.45),s_{0\langle o_{-1}\rangle}(0.55)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_0\rangle}(0),s_{1\langle o_0\rangle}(0),s_{1\langle o_0\rangle}(1)\}, \\ \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.4),s_{-1\langle o_{-1}\rangle}(0.6)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_{-2}\rangle}(0),s_{1\langle o_{-2}\rangle}(0.1),s_{2\langle o_{-1}\rangle}(0.6)\}, \\ \{s_{1\langle o_{-1}\rangle}(0),s_{1\langle o_{-1}\rangle}(0.3),s_{0\langle o_2\rangle}(0.7)\}, \\ \{s_{-2\langle o_2\rangle}(0),s_{-2\langle o_2\rangle}(0),s_{-2\langle o_2\rangle}(0.7)\} \end{bmatrix}$
A_2	$\begin{bmatrix} \{s_{-1\langle o_{-1}\rangle}(0.3),s_{1\langle o_{-2}\rangle}(0.3),s_{1\langle o_1\rangle}(0.4)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.5),s_{0\langle o_{-2}\rangle}(0.5)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(1)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.6),s_{1\langle o_{-2}\rangle}(0.4)\}, \\ \{s_{-1\langle o_0\rangle}(0.2),s_{0\langle o_{-1}\rangle}(0.2),s_{0\langle o_0\rangle}(0.4)\}, \\ \{s_{-2\langle o_0\rangle}(0),s_{-2\langle o_0\rangle}(0.6),s_{-1\langle o_{-1}\rangle}(0.4)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_{-2}\rangle}(0.4),s_{0\langle o_{-2}\rangle}(0.2),s_{1\langle o_{-1}\rangle}(0.4)\}, \\ \{s_{0\langle o_{-1}\rangle}(0),s_{0\langle o_{-1}\rangle}(0),s_{0\langle o_{-1}\rangle}(0.9)\}, \\ \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0.4),s_{-1\langle o_1\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_0\rangle}(0),s_{1\langle o_0\rangle}(0),s_{1\langle o_0\rangle}(0.5)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0.2),s_{-1\langle o_{-1}\rangle}(0.3),s_{0\langle o_{-2}\rangle}(0.5)\}, \\ \{s_{0\langle o_{-1}\rangle}(0),s_{0\langle o_{-1}\rangle}(0.3),s_{1\langle o_{-2}\rangle}(0.7)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.2),s_{1\langle o_{-1}\rangle}(0.8)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.8),s_{-1\langle o_1\rangle}(0.2)\}, \\ \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.4),s_{-1\langle o_0\rangle}(0.6)\} \end{bmatrix}$
A_3	$\begin{bmatrix} \{s_{0\langle o_2\rangle}(0),s_{1\langle o_{-2}\rangle}(0.3),s_{0\langle o_2\rangle}(0.7)\}, \\ \{s_{0\langle o_{-1}\rangle}(0),s_{0\langle o_{-1}\rangle}(0),s_{0\langle o_{-1}\rangle}(0.5)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0),s_{-1\langle o_{-2}\rangle}(0.6),s_{0\langle o_{-1}\rangle}(0.4)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-2\langle o_0\rangle}(0),s_{-2\langle o_0\rangle}(0),s_{-2\langle o_0\rangle}(0.7)\}, \\ \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0.3),s_{1\langle o_0\rangle}(0.7)\}, \\ \{s_{0\langle o_{-1}\rangle}(0),s_{0\langle o_{-1}\rangle}(0),s_{0\langle o_{-1}\rangle}(0.8)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_{-1}\rangle}(0),s_{1\langle o_{-1}\rangle}(0.6),s_{1\langle o_1\rangle}(0.4)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0),s_{-1\langle o_{-2}\rangle}(0.2),s_{-1\langle o_{-1}\rangle}(0.8)\}, \\ \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.55),s_{0\langle o_{-2}\rangle}(0.45)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-2\langle o_0\rangle}(0),s_{-2\langle o_0\rangle}(0.65),s_{1\langle o_{-1}\rangle}(0.35)\}, \\ \{s_{-1\langle o_0\rangle}(0),s_{0\langle o_{-1}\rangle}(0.4),s_{-1\langle o_0\rangle}(0.6)\}, \\ \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.75)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.6),s_{1\langle o_{-1}\rangle}(0.4)\}, \\ \{s_{0\langle o_{-2}\rangle}(0),s_{0\langle o_{-2}\rangle}(0.4),s_{0\langle o_{-1}\rangle}(0.6)\}, \\ \{s_{-2\langle o_1\rangle}(0.1),s_{-1\langle o_1\rangle}(0.3),s_{1\langle o_{-2}\rangle}(0.6)\} \end{bmatrix}$

Table 3. Standardized evaluation information of experts E_3

表 3. 专家 E_3 的标准化后的评价信息

	C_1	C_2	C_3	C_4	C_5
A_1	$\begin{bmatrix} \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.2),s_{-1\langle o_2\rangle}(0.8)\}, \\ \{s_{1\langle o_0\rangle}(0),s_{1\langle o_0\rangle}(0),s_{1\langle o_0\rangle}(0.65)\}, \\ \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_1\rangle}(0.35),s_{-1\langle o_0\rangle}(0.65)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_{-2}\rangle}(0),s_{1\langle o_{-2}\rangle}(0),s_{1\langle o_{-2}\rangle}(0.75)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0),s_{-1\langle o_{-2}\rangle}(0.4),s_{0\langle o_{-1}\rangle}(0.6)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.3),s_{0\langle o_{-1}\rangle}(0.45)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.7),s_{1\langle o_{-1}\rangle}(0.3)\}, \\ \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0.1),s_{0\langle o_{-1}\rangle}(0.9)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.75),s_{0\langle o_{-1}\rangle}(0.25)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0.5),s_{1\langle o_{-1}\rangle}(0.5)\}, \\ \{s_{-2\langle o_0\rangle}(0),s_{-2\langle o_0\rangle}(0.56),s_{-1\langle o_0\rangle}(0.44)\}, \\ \{s_{-2\langle o_{-1}\rangle}(0),s_{-2\langle o_{-1}\rangle}(0),s_{-2\langle o_{-1}\rangle}(0.5)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{0\langle o_{-2}\rangle}(0),s_{0\langle o_{-2}\rangle}(0.1),s_{1\langle o_{-1}\rangle}(0.6)\}, \\ \{s_{0\langle o_{-2}\rangle}(0),s_{0\langle o_{-2}\rangle}(0.1),s_{0\langle o_{-1}\rangle}(0.8)\}, \\ \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.2),s_{1\langle o_{-2}\rangle}(0.7)\} \end{bmatrix}$
A_2	$\begin{bmatrix} \{s_{1\langle o_{-2}\rangle}(0),s_{1\langle o_{-2}\rangle}(0.4),s_{0\langle o_1\rangle}(0.4)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.5),s_{1\langle o_{-2}\rangle}(0.5)\}, \\ \{s_{0\langle o_{-2}\rangle}(0),s_{0\langle o_{-1}\rangle}(0.3),s_{0\langle o_{-2}\rangle}(0.7)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_1\rangle}(0),s_{-1\langle o_1\rangle}(0.1),s_{2\langle o_{-1}\rangle}(0.9)\}, \\ \{s_{-2\langle o_0\rangle}(0.2),s_{-2\langle o_1\rangle}(0.2),s_{-1\langle o_{-2}\rangle}(0.4)\}, \\ \{s_{-2\langle o_0\rangle}(0),s_{-2\langle o_0\rangle}(0.3),s_{-1\langle o_{-1}\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0.7),s_{1\langle o_1\rangle}(0.3)\}, \\ \{s_{-2\langle o_0\rangle}(0),s_{-2\langle o_0\rangle}(0.2),s_{-1\langle o_{-1}\rangle}(0.2)\}, \\ \{s_{-1\langle o_2\rangle}(0),s_{-1\langle o_2\rangle}(0.4),s_{0\langle o_{-2}\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{0\langle o_{-1}\rangle}(0),s_{0\langle 0_{-1}\rangle}(0.3),s_{2\langle o_{-1}\rangle}(0.7)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0.2),s_{0\langle o_{-2}\rangle}(0.3),s_{0\langle o_{-1}\rangle}(0.5)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0),s_{-1\langle o_{-2}\rangle}(0),s_{-1\langle o_{-2}\rangle}(0.7)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_{-1}\rangle}(0),s_{2\langle o_{-1}\rangle}(0.3),s_{1\langle o_{-1}\rangle}(0.6)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.7),s_{1\langle o_0\rangle}(0.3)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0),s_{-1\langle o_{-1}\rangle}(0.4),s_{1\langle o_{-1}\rangle}(0.6)\} \end{bmatrix}$
A_3	$\begin{bmatrix} \{s_{0\langle o_{-2}\rangle}(0),s_{0\langle o_{-2}\rangle}(0.6),s_{1\langle o_0\rangle}(0.4)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0),s_{-1\langle o_{-2}\rangle}(0.2),s_{0\langle o_{-2}\rangle}(0.7)\}, \\ \{s_{-2\langle o_2\rangle}(0),s_{-2\langle o_2\rangle}(0.6),s_{-1\langle o_{-2}\rangle}(.04)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0.8)\}, \\ \{s_{2\langle o_{-1}\rangle}(0),s_{2\langle o_{-1}\rangle}(0),s_{2\langle o_{-1}\rangle}(0.7)\}, \\ \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.45)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_0\rangle}(0.25),s_{0\langle o_{-1}\rangle}(0.3),s_{1\langle o_{-2}\rangle}(0.45)\}, \\ \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0.15),s_{1\langle o_{-1}\rangle}(0.85)\}, \\ \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.5),s_{0\langle o_{-2}\rangle}(0.5)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_{-2}\rangle}(0),s_{1\langle o_0\rangle}(0.1),s_{1\langle o_{-2}\rangle}(0.75)\}, \\ \{s_{-1\langle o_0\rangle}(0),s_{-1\langle o_0\rangle}(0.5),s_{1\langle o_{-2}\rangle}(0.5)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0),s_{-1\langle o_{-2}\rangle}(0.4),s_{0\langle o_{-1}\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.3),s_{1\langle o_{-1}\rangle}(0.3)\}, \\ \{s_{-1\langle o_0\rangle}(0.2),s_{0\langle o_{-2}\rangle}(0.2),s_{1\langle o_{-2}\rangle}(0.6)\}, \\ \{s_{-2\langle o_1\rangle}(0),s_{-2\langle o_1\rangle}(0.3),s_{0\langle o_{-1}\rangle}(0.6)\} \end{bmatrix}$

步骤 2：分析 3 名专家的社会网络关系，他们之间的关系强度矩阵表示为

$$R=\begin{pmatrix} 1 & 0.3 & 0.8 \\ 0.5 & 1 & 0.7 \\ 0.8 & 0.6 & 1 \end{pmatrix}$$

其中矩阵中对应元素值越大，表示他们之间的联系越紧密。利用式(16)，计算 3 名专家的一致程度，进而构造得到信任度矩阵，将每位专家的总信任度标准化后，得到专家主观权重为

$$W_{1e}=[0.3368,0.2879,0.3735]$$

设各专家以及专家相互关联的特征函数值为 $u(E_1)=0.35$，$u(E_2)=0.2$，$u(E_3)=0.25$，$u(E_1,E_2)=0.6$，$u(E_1,E_3)=0.7$，$u(E_2,E_3)=0.5$，$u(E_1,E_2,E_3)=1$，各专家的 Shapley 值向量为 $W_{2e}=[0.425,0.25,0.325]$。则综合权重为 $W_e=[0.3809,0.2689,0.3492]$。

步骤 3：根据专家权重，利用 $PDHLTSF\text{-}DWA_w^\theta$ 算子，将评价信息聚合，得到群决策矩阵。

步骤 4：根据 5 位不同领域的专家 e_i 对安全风险评估中的属性重要度进行主观评价，见表 4，利用 IDOCRIW 方法确定属性权重。首先根据熵权法的计算步骤，得到各属性的权重为 $W_1=[0.1898,0.2511,0.1478,0.2304,0.1809]$；然后，由 CILOS 方法，得到 CILOS 方法下的权重 $W_2=[0.2158,0.1942,0.1886,0.1937,0.2077]$；通过式(26)，得到综合属性权重

$\varpi = [0.2050, 0.2441, 0.1395, 0.2234, 0.188]$。根据综合权重，利用 $PDHLTF\text{-}DWG_w^\theta$ 算子对群决策矩阵进行聚合，得到综合决策，见表 5。

Table 4. Experts' evaluation information on attribute importance
表 4. 专家对于属性重要度的评价信息

	C_1	C_2	C_3	C_4	C_5
e_1	$\begin{bmatrix} \{s_{1\langle o_0\rangle}(0.6)\},\{s_{1\langle o_{-2}\rangle}(0.8)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0.2)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_0\rangle}(0.8)\},\{s_{2\langle o_{-2}\rangle}(1)\}, \\ \{s_{1\langle o_{-1}\rangle}(0.8)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{0\langle o_{-2}\rangle}(1)\},\{s_{0\langle o_{-2}\rangle}(0.6),s_{0\langle o_{-1}\rangle}(0.4)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0.8)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{0\langle o_{-1}\rangle}(0.4),s_{2\langle o_{-1}\rangle}(0.6)\}, \\ \{s_{-2\langle o_{-1}\rangle}(0.8)\},\{s_{-1\langle o_{-2}\rangle}(0.2)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{0\langle o_{-2}\rangle}(1)\},\{s_{1\langle o_{-2}\rangle}(0.8)\}, \\ \{s_{1\langle o_0\rangle}(0.6)\} \end{bmatrix}$
e_2	$\begin{bmatrix} \{s_{0\langle o_{-2}\rangle}(0.8)\},\{s_{-1\langle o_2\rangle}(0.7)\}, \\ \{s_{2\langle o_{-2}\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_{-1}\rangle}(0.8)\},\{s_{1\langle o_{-1}\rangle}(0.6)\}, \\ \{s_{1\langle o_{-1}\rangle}(0.5)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_0\rangle}(0.8)\},\{s_{-1\langle o_0\rangle}(0.5)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{0\langle o_{-1}\rangle}(0.6)\},\{s_{-2\langle o_1\rangle}(1)\}, \\ \{s_{-1\langle o_0\rangle}(0.3)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_0\rangle}(0.5)\},\{s_{1\langle o_0\rangle}(0.8)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0.6)\} \end{bmatrix}$
e_3	$\begin{bmatrix} \{s_{-1\langle o_0\rangle}(1)\},\{s_{-1\langle o_1\rangle}(0.7)\}, \\ \{s_{0\langle o_2\rangle}(0.3)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_{-1}\rangle}(1)\},\{s_{-1\langle o_1\rangle}(0.8)\}, \\ \{s_{0\langle o_0\rangle}(0.3)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{0\langle o_{-2}\rangle}(0.8)\},\{s_{1\langle o_0\rangle}(0.2)\}, \\ \{s_{1\langle o_{-2}\rangle}(0.5)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_{-1}\rangle}(0.8)\},\{s_{1\langle o_{-2}\rangle}(1)\}, \\ \{s_{-1\langle o_{-2}\rangle}(0.5)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_1\rangle}(0.5)\},\{s_{-1\langle o_{-2}\rangle}(1)\}, \\ \{s_{0\langle o_{-1}\rangle}(0.8)\} \end{bmatrix}$
e_4	$\begin{bmatrix} \{s_{1\langle o_{-1}\rangle}(1)\},\{s_{-2\langle o_1\rangle}(1)\}, \\ \{s_{1\langle o_0\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_2\rangle}(0.3)\},\{s_{1\langle o_{-1}\rangle}(0.2)\}, \\ \{s_{1\langle o_{-2}\rangle}(0.6)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_2\rangle}(1)\},\{s_{1\langle o_{-2}\rangle}(0.95)\}, \\ \{s_{1\langle o_0\rangle}(0.55)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_{-1}\rangle}(1)\},\{s_{1\langle o_{-1}\rangle}(1)\}, \\ \{s_{0\langle o_{-2}\rangle}(0.8)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_1\rangle}(1)\},\{s_{1\langle o_0\rangle}(0.3)\}, \\ \{s_{-1\langle o_0\rangle}(1)\} \end{bmatrix}$
e_5	$\begin{bmatrix} \{s_{2\langle o_{-1}\rangle}(0.8)\},\{s_{-1\langle o_{-1}\rangle}(0.7)\}, \\ \{s_{1\langle o_{-2}\rangle}(0.3)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-2\langle o_2\rangle}(1)\},\{s_{1\langle o_1\rangle}(1)\}, \\ \{s_{0\langle o_{-1}\rangle}(0.5)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-1\langle o_1\rangle}(0.8)\},\{s_{-2\langle o_1\rangle}(0.5)\}, \\ \{s_{0\langle o_{-1}\rangle}(0.65)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{-2\langle o_{-1}\rangle}(1)\},\{s_{2\langle o_{-1}\rangle}(0.8)\}, \\ \{s_{-2\langle o_{-2}\rangle}(0.3)\} \end{bmatrix}$	$\begin{bmatrix} \{s_{1\langle o_0\rangle}(1)\},\{s_{0\langle o_{-2}\rangle}(0.6),s_{-1\langle o_{-1}\rangle}(0.4)\}, \\ \{s_{-1\langle o_{-1}\rangle}(0.6)\} \end{bmatrix}$

步骤 5：由式(11)，得到各个景点的得分值 $S(A) = [-0.0821, 0.0527, -0.0808]$，根据排序规则，得到最优排序 $A_2 \succ A_3 \succ A_1$。

Table 5. Final aggregation results
表 5. 最终聚合结果

	聚合结果	综合评价值
A_1	$\begin{bmatrix} \{s_{-1\langle o_{-1.84}\rangle}(0.01),s_{-1\langle o_{-1.83}\rangle}(0.32),s_{-1\langle o_{-0.48}\rangle}(0.67)\}, \\ \{s_{-1\langle o_{-1.643}\rangle}(0),s_{-1\langle o_{-1.641}\rangle}(0.25),s_{0\langle o_{-1.04}\rangle}(0.75)\}, \\ \{s_{-1\langle o_{-1.69}\rangle}(0),s_{-1\langle o_{-1.61}\rangle}(0.243),s_{-1\langle o_{-1.21}\rangle}(0.756)\} \end{bmatrix}$	−0.0821
A_2	$\begin{bmatrix} \{s_{-1\langle o_{-1.55}\rangle}(0.06),s_{-1\langle o_{-1.49}\rangle}(0.34),s_{0\langle o_{-1.3}\rangle}(0.6)\}, \\ \{s_{-1\langle o_{-1.96}\rangle}(0.08),s_{-1\langle o_{1.93}\rangle}(0.4),s_{-1\langle o_{-0.44}\rangle}(0.52)\}, \\ \{s_{-1\langle o_{-1.64}\rangle}(0),s_{-1\langle o_{-1.57}\rangle}(0.31),s_{1\langle o_{-1.63}\rangle}(0.69)\} \end{bmatrix}$	0.0527
A_3	$\begin{bmatrix} \{s_{-1\langle o_{-1.62}\rangle}(0.02),s_{-1\langle o_{-1.64}\rangle}(0.33),s_{-1\langle o_{-1.58}\rangle}(0.65)\}, \\ \{s_{-1\langle o_{-1.84}\rangle}(0.01),.s_{-1\langle o_{-1.8}\rangle}(0.25),s_{-1\langle o_{-1.56}\rangle}(0.74)\}, \\ \{s_{-1\langle o_{-1.478}\rangle}(0.03),s_{-1\langle o_{-1.48}\rangle}(0.31),s_{-1\langle o_{-1.4}\rangle}(0.66)\} \end{bmatrix}$	−0.0808

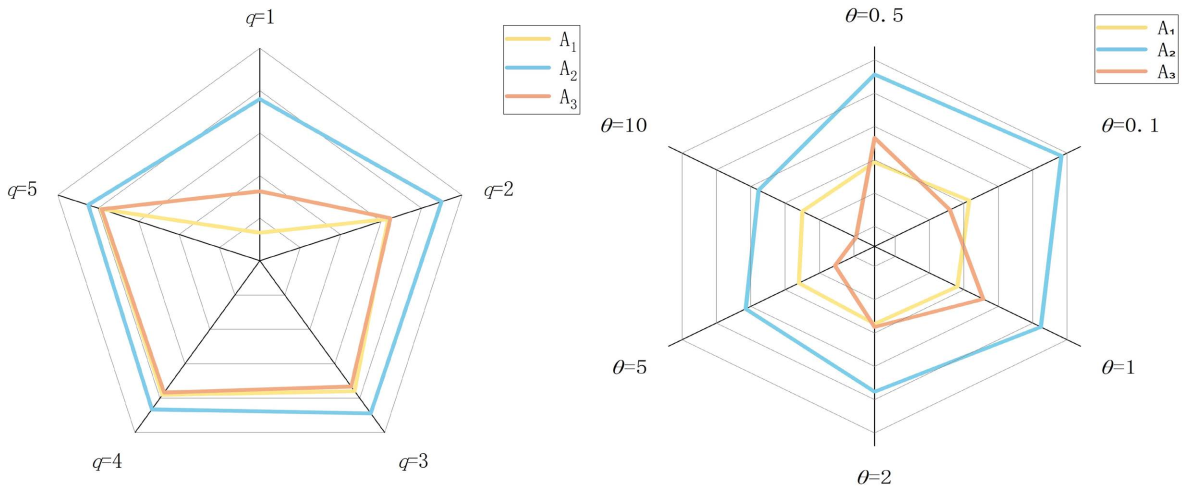
结果显示，景点 A_2 的安全系数高于其余两个景点，而景点 A_1 安全系数最低，因此景点 A_1 的管理者应对该地的安全设施或管理进行检查和整改。

6.2. 敏感性分析

在上文提出的 $PDHLTSF\text{-}DWA_w^\theta$ 算子中，除了概率双层语言 T 球形模糊集的参数 q 以及聚合权重 w，Dombi 范数中的 θ 也是影响聚合结果的一个因素。而本文提出的聚合算子，对于参数 θ 具有单调性，从一方面也能反映出决策者对于决策的风险态度。θ 越大，说明决策者更偏向在承担风险下做出选择。如果

决策者更偏向平衡风险，一般取 $\theta \in (0,2]$；如果决策者更看重项目的潜在高收益和巨大的市场机会，一般取 $\theta \in (5,+\infty)$。因此，可以通过调整参数值来反映决策者的风险承受能力与实际需求。

图 2 给出了当 w 取实例中的值，并固定 q,θ 中的一个参数，另一个参数取不同值时，各备选方案的得分函数值。容易发现，当参数变化时，A_1 的得分函数值始终最大，即安全系数最大的始终是 A_1，反映出参数变化对备选方案最优选择具有稳健性，验证了所提方法具有一定的有效性。



(a) q 不同取值下的综合评价值　　(b) θ 不同取值下的综合评价值

Figure 2. The comprehensive evaluation value under different parameter changes
图 2. 不同参数变化下的综合评价值

6.3. 对比分析

为了进一步验证本文提出方法的合理性和有效性，本小节通过将所提方法的计算结果与现有其他方法的计算结果进行比较。由于本文引入 PDHLTSFS，现有的一些决策方法无法直接处理这类数据，因此考虑在 PDHLTSFS 的某些特殊情形下对于所提模糊集的合理性进行验证。

1) 当 $G_s(p)$ 为空集时，若 $H_s(p), I_s(p)$ 中只有一个双层语言术语，且对应概率都为 1 时，PDHLTSFS 退化为直觉双层语言模糊集。

2) 当 $H_s(p), G_s(p), I_s(p)$ 中的双层语言术语用 0 到 1 之间的实数替代时，PDHLTSFS 退化为 T 球形犹豫概率模糊集。

因此本文将所提的实例分析结果与利用文献[22] [23]中的决策方法进行比较。

实例中决策者对于每个属性是带有一定的犹豫度，而在直觉双层语言模糊集(Intuitionistic Double Hierarchy Linguistic Fuzzy Sets, IDHLFS)中，隶属度和非隶属度都只用一个双层语言术语表示，为了全面合理地对比本文所提方法与文献，分为两种策略下进行对比，即乐观策略和悲观策略。乐观策略是指将 PDHLTSFS 退化为 IDHLFS 时，取 PDHLTSFE 中 $H_s(p)$ 语言术语最大的元素作为 IDHLFS 的隶属度，取 $I_s(p)$ 中语言术语最小的元素作为 IDHLFS 的非隶属度；悲观策略则相反。

为了减小其余因素对于对比结果的影响，在对比实验中决策者的权重和属性的权重均为实例中的取值。对比结果如表 6 所示。

从表中看出，安全性最高的景点都是 A_2，与本文所提方法得出的结果一致。但是在参数 $\eta = 2, \gamma = 2,5$ 时，排序的结果区分度不高，可能是因为在文献[22]中 IDHLFS 忽略了中立隶属度以及概率信息。另一方

面，文献[23]仅用定量信息表征评价信息，一定程度上缺失了一部分定性信息，也进一步证明了本文方法的优越性。

Table 6. The ranking results of existing methods
表 6. 现有方法的排序结果

方法		参数值	排序结果
IDHLHOWA [22]	乐观策略	$\gamma=1$	$A_2 \succ A_3 \succ A_1$
		$\gamma=2$	$A_2 \approx A_3 \succ A_1$
		$\gamma=5$	$A_2 \approx A_3 \succ A_1$
	悲观策略	$\gamma=1$	$A_2 \succ A_3 \succ A_1$
		$\gamma=2$	$A_2 \approx A_3 \succ A_1$
		$\gamma=5$	$A_2 \approx A_3 \succ A_1$
T-SHPFS TODIM-TPZSG Method [23]		$\eta=0.5$	$A_2 \succ A_3 \succ A_1$
		$\eta=1$	$A_2 \succ A_3 \succ A_1$
		$\eta=2$	$A_2 \succ A_3 \approx A_1$
		$\eta=5$	$A_2 \succ A_3 \succ A_1$

7. 结论

本文综合概率语言球形集和双层语言术语集的特性，提出概率双层语言 T 球形模糊集。定义了概率双层语言 T 球形模糊集的得分函数、精确函数以及 Hamming 距离，并基于 Dombi 范数的运算规则提出了 $PDHLTSF\text{-}DWA_w^\theta$，并且研究其满足的一些性质。然后，给出一种构建了基于社交网络的信任度和一致性的决策者权重确定方法。其次，利用 IDOCRIW 方法得到属性的客观权重，提出基于概率双层语言 T 球形模糊集的多属性决策方法。最后，将该方法应用于一个网红景点安全性评估问题，并与其他方法进行对比，证明了该方法的合理性与有效性。

尽管本文研究了概率双层语言 T 球形模糊集的 Dombi 运算和聚合算子，但仍然存在一些局限。如对概率双层语言 T 球形模糊集规范化时，直接将最小的语言术语添加到模糊集中，这可能导致决策者的评价信息无法得到精确表达，从而导致决策结果出现一定偏差；对于一些更为复杂的实际问题，提出的加权平均算子在一定程度无法反映出属性间的相关性。未来的工作可以对于以上问题提出新的解决方法或者在决策者和属性的权重确定方面探索出更符合实际的计算方法。

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