Advances in Applied Mathematics 应用数学进展, 2025, 14(12), 183-189
Published Online December 2025 in Hans. https://www.hanspub.org/journal/aam
https://doi.org/10.12677/aam.2025.1412498

一维Navier-Stokes-Smoluchowski系统的自由边界问题

丁茵菲，黄金锐*

五邑大学数学与计算科学学院，广东 江门

收稿日期：2025年11月8日；录用日期：2025年12月1日；发布日期：2025年12月10日

摘 要

研究了一类可压缩粒子流体耦合模型的一维自由边界问题，借助拉格朗日坐标变换建立解的基本能量不等式。

关键词

Navier-Stokes-Smoluchowski系统，自由边界问题，基本能量不等式

Free Boundary Problem for One-Dimensional Navier-Stokes-Smoluchowski Systems

Yinfei Ding, Jinrui Huang*

School of Mathematics and Computational Science, Wuyi University, Jiangmen Guangdong

Received: November 8, 2025; accepted: December 1, 2025; published: December 10, 2025

Abstract

The one-dimensional free boundary problem of a particle-fluid interation model is studied in this paper, and the basic energy law is established by Lagrange coordinate transformation.

*通讯作者。

文章引用: 丁茵菲, 黄金锐. 一维 Navier-Stokes-Smoluchowski 系统的自由边界问题[J]. 应用数学进展, 2025, 14(12): 183-189. DOI: 10.12677/aam.2025.1412498

Keywords

Navier-Stokes-Smoluchowski System, Free Boundary Problem, Basic Energy Law

1. 模型与预备知识

在当代多相流体力学的研究中，颗粒与流体之间的交互作用已成为热点研究方向之一。其应用广泛涉及能源、化工、生物医药等关键领域，成为关联基础科学前沿与重大工程需求的桥梁。本文考虑一类流体粒子相互作用的 Navier-Stokes-Smoluchowski 模型[1]-[6]：

$$\begin{cases}\rho_t+(\rho u)_x=0,\\ (\rho u)_t+(\rho u^2)_x+(P(\rho,\eta))_x+(\rho+\eta)\Phi_x+\eta_x-(2\mu u_x)_x-(\lambda u_x)_x=0,\\ \eta_t+(\eta(u-\Phi_x))_x-\eta_{xx}=0.\end{cases}\tag{1.1}$$

其中 $(x,t)\in(a(t),b(t))\times[0,T]$。$\rho$ 为流体密度，Φ 为外势力(只与重力、浮力相关)，u 为流体速度，η 为粒子密度，$\tilde{P}=\tilde{P}(\rho,\eta)=A\rho^\gamma+\eta$ 为正压力，为简单起见，假定常数 $\gamma>1$ 和 $A>0$，粘性系数 μ 和满 λ 足 $\mu>0,2\mu+\lambda\geq 0$.考虑系统满足初始条件：

$$(\rho,u,\eta)\big|_{t=0}=(\rho_0(x),u_0(x),\eta_0(x)).\tag{1.2}$$

其中 $x\in[a(0),b(0)]$。满系统满足速度场的无滑移边界条件以及粒子密度的无流边界条件：

$$u(x,t)\big|_{x=a(t),b(t)}=0,(\eta_x+\eta\Phi_x)\big|_{x=a(t),b(t)}=0.\tag{1.3}$$

上述模型数学理论的研究始于 Ballew 以及其合作者[1] [2]，建立了高维弱解的整体存在性以及低分层极限的收敛性。进而，文献[3]-[5]建立了高维强解的局部适定性以及小条件整体适定性。对于一维固定边界问题，文献[6]建立了经典解的整体适定性。而关于该模型的自由边界问题，目前而言结果较少。

借助拉格朗日坐标变换，可以将自由边界转化为固定边界 $(y,\tau)\in[0,1]\times[0,T]$。考虑拉格朗日坐标变换如下：

$$y=\int_0^x\rho(\xi,t)\mathrm{d}\xi,t=\tau.\tag{1.4}$$

$$y_x=\rho(x,t).\tag{1.5}$$

根据拉格朗日变换，方程组(1.1)可改写为：

$$\begin{cases}\rho_\tau+\rho^2u_y=0,\\ u_\tau+(P(\rho))_y+\eta_y=(2\mu+\lambda)(\rho u_y)_y-(\rho+\eta)\Phi_y,\\ \eta_\tau+\rho\eta u_y-\rho(\rho\eta_y)_y-\rho(\rho\eta\Phi_y)_y=0.\end{cases}\tag{1.6}$$

考虑流体、粒子质量守恒，不妨设：

$$\int_0^1\rho(\xi,t)\mathrm{d}\xi=1,\int_0^1\eta(\xi,t)\mathrm{d}\xi=1.\tag{1.7}$$

系统的初始条件变为：

$$\left(\rho,u,\eta\right)\big|_{\tau=0}=\left(\rho_0,u_0,\eta_0\right),\ y\in\left[0,1\right]. \tag{1.8}$$

边界条件变为：

$$u\left(y,\tau\right)=\left(\rho\eta_y+\rho\eta\Phi_y\right)\left(y,\tau\right)=0,\ y=0,1,\ \tau\in\left[0,T\right]. \tag{1.9}$$

考虑粒子密度满足结构性条件：

$$\int_0^\tau \sup_{x\in I}\left|\eta-\eta_s\right|^2\mathrm{d}\sigma\le C,\ \tau\in\left[0,T\right]. \tag{1.10}$$

其中，稳态解$\left(\rho_s,\eta_s\right)$满足稳态方程：

$$\begin{cases}P_y\left(\rho_s\right)=-\rho_s\Phi_y,\\ \left(\eta_s\right)_y=-\eta_s\Phi_y.\end{cases} \tag{1.11}$$

定义依赖于压力项与稳态解两个辅助函数：

$$\begin{aligned}&G_1\left(\rho\right)=\int_{\rho_s}^{\rho}\int_{\rho_s}^{r}\frac{P'\left(s\right)}{s}\mathrm{d}s\mathrm{d}r=\rho\int_{\rho_s}^{\rho}\frac{P\left(s\right)-P\left(\rho_s\right)}{s^2}\mathrm{d}s,\\ &G_2\left(\eta\right)=\eta\int_{\eta_s}^{\eta}\frac{s-\eta_s}{s^2}\mathrm{d}s.\end{aligned} \tag{1.12}$$

其中，$G_1\left(\rho_s\right)=0$，且

$$G_1'\left(\rho\right)=\frac{G_1\left(\rho\right)+P\left(\rho\right)-P\left(\rho_s\right)}{\rho},G_1'\left(\rho_s\right)=0,G_1''\left(\rho\right)=\frac{P'\left(\rho\right)}{\rho}. \tag{1.13}$$

改写$(1.6)_2$为

$$u_\tau+\rho\left(\frac{P_y\left(\rho\right)}{\rho}-\frac{P_y\left(\rho_s\right)}{\rho_s}\right)+\eta\left(\frac{\eta_y}{\eta}-\frac{\left(\eta_s\right)_y}{\eta_s}\right)=\left(2\mu+\lambda\right)\left(\rho u_y\right)_y. \tag{1.14}$$

1989 年，Mari Okada [7]探讨了一维可压缩 Navier-Stokes 方程自由边界问题的全局解的存在性以及长时间行为；1992 年，Okada 和 Makino [8]考虑球对称情形的自由边界问题，得到全局弱解的存在性。2013 年，黄金锐等人[9]证明了初始密度真空跳跃连接条件下，向列型液晶 Ericksen-Leslie 简化系统全局经典解的存在唯一性；2015 年，黄金锐和丁时进[10]进一步得到了初始真空连续连接条件下全局弱解的存在性。基于对已有文献的研究，本文考虑一维 Navier-Stokes-Smoluchowski 系统耦合模型的自由边界问题，分析系统的基本能量不等式。

2. 主要结果

主要问题是建立系统(1.6)，(1.8)，(1.9)，(1.10)的基本能量率。

定理 1：假定$\left(\rho,u,\eta\right)$为系统(1.6)，(1.8)，(1.9)，(1.10)的全局强解，则存在常数$C\left(E_0,T\right)$，使得：

$$\begin{aligned}&\int_I\left[\frac{\left|u\right|^2}{2}+\frac{G_1\left(\rho\right)}{\rho}+\frac{G_2\left(\eta\right)}{\rho}+\frac{\eta\ln\eta}{\rho}+\frac{2\eta\Phi}{\rho}\right]\left(\cdot,\tau\right)\mathrm{d}y+\frac{2\mu+\lambda}{2}\int_0^T\int_I\rho u_y^2\mathrm{d}y\mathrm{d}\sigma\\ &+\int_0^T\int_I 2\rho\left|\frac{\eta_y}{\sqrt{\eta}}+\sqrt{\eta}\Phi_y\right|^2\mathrm{d}y\mathrm{d}\sigma\le C\left(E_0,T\right).\end{aligned} \tag{2.1}$$

其中，$\tau\in\left[0,T\right]$以及$E_0=\int_I\left[\frac{\left|u_0\right|^2}{2}+\frac{G_1\left(\rho_0\right)}{\rho_0}+\frac{G_1\left(\eta_0\right)}{\rho_0}+\frac{\eta_0\ln\eta_0}{\rho_0}+\frac{2\eta_0\Phi}{\rho_0}\right]\mathrm{d}y$.

证明：证明主要分成三步。

步骤一：用 u 点乘(1.14)并在空间 I 上进行积分，分部积分并结合边界条件(1.9)，可得：

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I \frac{|u|^2}{2}\mathrm{d}y+\int_I \rho u\left(\frac{P_y(\rho)}{\rho}-\frac{P_y(\rho_s)}{\rho_s}\right)\mathrm{d}y+\int_I \eta u\left(\frac{\eta_y}{\eta}-\frac{\eta_{sy}}{\eta_s}\right)\mathrm{d}y=-(2\mu+\lambda)\int_I \rho u_y^2\mathrm{d}y. \tag{2.2}$$

根据定义(1.12)，有

$$G_1'(\rho)=\int_{\rho_s}^{\rho}\frac{P'(s)}{s}\mathrm{d}s=\int_{\rho_s}^{\rho}A\gamma s^{\gamma-2}\mathrm{d}s=\frac{A\gamma}{\gamma-1}\left(\rho^{\gamma-1}-\rho_s^{\gamma-1}\right). \tag{2.3}$$

从而利用分部积分以及(1.6)$_1$，可得

$$\begin{aligned}
&\int_I \rho u\left(\frac{P_y(\rho)}{\rho}-\frac{P_y(\rho_s)}{\rho_s}\right)\mathrm{d}y\\
&=A\gamma\int_I \rho u\left(\rho^{\gamma-2}\rho_y-\rho_s^{\gamma-2}(\rho_s)_y\right)\mathrm{d}y=\int_I \rho u\left(G_1'(\rho)\right)_y\mathrm{d}y\\
&=-\int_I (\rho u)_y G_1'(\rho)\mathrm{d}y=\int_I\left(\frac{\rho_\tau}{\rho}-u\rho_y\right)G_1'(\rho)\mathrm{d}y\\
&=\int_I \frac{\rho_\tau}{\rho}G_1'(\rho)\mathrm{d}y+\int_I u_y G_1(\rho)\mathrm{d}y=\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\frac{G_1(\rho)}{\rho}\mathrm{d}y.
\end{aligned} \tag{2.4}$$

另外，由于

$$\begin{aligned}
&G_2(\eta)=\eta\left(\ln s+\eta_s\frac{1}{s}\right)\Big|_{\eta_s}^{\eta}=\eta\ln\frac{\eta}{\eta_s}+\eta_s-\eta,\\
&G_2'(\eta)=\frac{G_2(\eta)+\eta-\eta_s}{\eta}=\ln\frac{\eta}{\eta_s},
\end{aligned} \tag{2.5}$$

可得

$$\begin{aligned}
\int_I \eta u\left(\frac{\eta_y}{\eta}-\frac{(\eta_s)_y}{\eta_s}\right)\mathrm{d}y&=\int_I \eta u\left(\ln\frac{\eta}{\eta_s}\right)_y\mathrm{d}y\\
&=\int_I \eta u\left(G_2'(\eta)\right)_y\mathrm{d}y\\
&=-\int_I (\eta u)_y G_2'(\eta)\mathrm{d}y.
\end{aligned} \tag{2.6}$$

再次应用分部积分、(1.6)$_3$、边界条件(1.9)以及 $G_2''(\eta)=\frac{1}{\eta}$，可得：

$$\begin{aligned}
&\int_I \eta u\left(\frac{\eta_y}{\eta}-\frac{\eta_{sy}}{\eta_s}\right)\mathrm{d}y=-\int_I(\eta u)_y G_2'(\eta)\mathrm{d}y\\
&=\int_I\left[\frac{\eta_\tau}{\rho}-(\rho\eta_y)_y-(\rho\eta\Phi_y)_y\right]G_2'(\eta)\mathrm{d}y-\int_I u\eta_y G_2'(\eta)\mathrm{d}y\\
&=\int_I\frac{\left(G_2(\eta)\right)_\tau}{\rho}\mathrm{d}y+\int_I\left(\rho\eta_y+\rho\eta\Phi_y\right)\left(G_2'(\eta)\right)_y\mathrm{d}y-\int_I u\eta_y G_2'(\eta)\mathrm{d}y\\
&=\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\frac{G_2(\eta)}{\rho}\mathrm{d}y+\int_I\frac{G_2(\eta)}{\rho^2}\rho_\tau\mathrm{d}y+\int_I\left(\rho\eta_y+\rho\eta\Phi_y\right)G_2''(\eta)\eta_y\mathrm{d}y-\int_I u\left(G_2(\eta)\right)_y\mathrm{d}y\\
&=\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\frac{G_2(\eta)}{\rho}\mathrm{d}y-\int_I G_2(\eta)u_y\mathrm{d}y+\int_I\left(\rho\eta_y+\rho\eta\Phi_y\right)\frac{\eta_y}{\eta}\mathrm{d}y+\int_I u_y G_2(\eta)\mathrm{d}y\\
&=\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\frac{G_2(\eta)}{\rho}\mathrm{d}y+\int_I\frac{\rho\eta_y^2}{\eta}\mathrm{d}y+\int_I\rho\Phi_y\eta_y\mathrm{d}y.
\end{aligned} \tag{2.7}$$

综合上述(2.2)~(2.5)，可得：

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\left[\frac{|u|^2}{2}+\frac{G_1(\rho)}{\rho}+\frac{G_1(\eta)}{\rho}\right]\mathrm{d}y+(2\mu+\lambda)\int_I\rho u_y^2\mathrm{d}y+\int_I\frac{\rho}{\eta}\eta_y^2\mathrm{d}y+\int_I\rho\Phi_y\eta_y\mathrm{d}y=0. \tag{2.8}$$

步骤二：将 $\frac{1}{\rho}(\ln\eta+1)$ 乘以$(1.6)_3$并在空间 I 上积分可得：

$$\begin{aligned}&\int_I\frac{\eta_\tau}{\rho}(\ln\eta+1)\mathrm{d}y+\int_I\eta u_y(\ln\eta+1)\mathrm{d}y-\int_I(\rho\eta_y)_y(\ln\eta+1)\mathrm{d}y\\&-\int_I(\rho\eta\Phi_y)_y(\ln\eta+1)\mathrm{d}y=0.\end{aligned} \tag{2.9}$$

由于 $(\ln\eta+1)_y=\frac{\eta_y}{\eta}$，对上式分部积分并利用$(1.6)_1$以及(1.9)，整理后得到：

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\frac{\eta\ln\eta}{\rho}\mathrm{d}y+\int_I\left(\rho\frac{\eta_y^2}{\eta}+\rho\eta_y\Phi_y\right)\mathrm{d}y=-\int_I\eta u_y\mathrm{d}y. \tag{2.10}$$

结合(2.8)与(2.10)式得到：

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\left[\frac{|u|^2}{2}+\frac{G_1(\rho)}{\rho}+\frac{G_1(\eta)}{\rho}+\frac{\eta\ln\eta}{\rho}\right]\mathrm{d}y+(2\mu+\lambda)\int_I\rho u_y^2\mathrm{d}y\\&+2\int_I\frac{\rho}{\eta}\eta_y^2\mathrm{d}y+2\int_I\rho\Phi_y\eta_y\mathrm{d}y=-\int_I\eta u_y\mathrm{d}y.\end{aligned} \tag{2.11}$$

另外，将 $\frac{\Phi}{\rho}$ 乘以(1.6)的第三条式子并在空间 I 上积分以及分部积分得到：

$$\begin{aligned}&\int_I\left[\eta_\tau\frac{\Phi}{\rho}+\eta u_y\Phi\right]\mathrm{d}y-\int_I(\rho\eta_y)_y\Phi\mathrm{d}y-\int_I(\rho\eta\Phi_y)_y\Phi\mathrm{d}y\\&=\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\frac{\eta\Phi}{\rho}\mathrm{d}y+\int_I\rho\eta_y\Phi_y\mathrm{d}y+\int_I\rho\eta\Phi_y^2\mathrm{d}y=0.\end{aligned} \tag{2.12}$$

结合上述(2.11)与(2.12)式可得：

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I\left[\frac{|u|^2}{2}+\frac{G_1(\rho)}{\rho}+\frac{G_1(\eta)}{\rho}+\frac{\eta\ln\eta}{\rho}+\frac{2\eta\Phi}{\rho}\right]\mathrm{d}y+(2\mu+\lambda)\int_I\rho u_y^2\mathrm{d}y\\&+2\int_I\rho\left|\frac{\eta_y}{\sqrt{\eta}}+\sqrt{\eta}\Phi_y\right|^2\mathrm{d}y=-\int_I\eta u_y\mathrm{d}y.\end{aligned} \tag{2.13}$$

步骤三：下文证明(2.11)的右端项满足如下不等式：

$$-\int_I\eta u_y\mathrm{d}y\le\frac{2\mu+\lambda}{2}\int_I\rho u_y^2\mathrm{d}y+C\sup_{x\in I}(\eta-\eta_s)^2+C. \tag{2.14}$$

证明：通过引入稳态解，可得

$$-\int_I\eta u_y\mathrm{d}y=-\int_I(\eta-\eta_s)u_y\mathrm{d}y-\int_I\eta_s u_y\mathrm{d}y=-\int_I\frac{\eta-\eta_s}{\sqrt{\rho}}\sqrt{\rho}u_y\mathrm{d}y-\int_I\eta_s u_y\mathrm{d}y. \tag{2.15}$$

应用 Hölder 不等式，Cauchy 不等式，可得：

$$-\int_I \eta u_y \mathrm{d}y \le \left(\int_I \frac{|\eta-\eta_s|^2}{\rho}\mathrm{d}y\right)^{\frac{1}{2}}\left(\int_I \rho u_y^2 \mathrm{d}y\right)^{\frac{1}{2}} - \int_I \frac{\eta_s}{\sqrt{\rho}}\sqrt{\rho}u_y \mathrm{d}y$$
$$\le \left(\int_I \frac{|\eta-\eta_s|^2}{\rho}\mathrm{d}y\right)^{\frac{1}{2}}\left(\int_I \rho u_y^2 \mathrm{d}y\right)^{\frac{1}{2}} + \frac{2\mu+\lambda}{4}\left(\int_I \rho u_y^2 \mathrm{d}y\right) + C\int_I \frac{1}{\rho}\mathrm{d}y. \tag{2.16}$$

根据 $\rho_\tau + \rho^2 u_y = 0$，可得：

$$\int_I \int_0^\tau \left(\frac{1}{\rho}\right)_\tau \mathrm{d}\sigma \mathrm{d}y = \int_I \int_0^\tau u_y \mathrm{d}\sigma \mathrm{d}y,$$
$$\int_I \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)\mathrm{d}y = \int_0^\tau \int_I u_y \mathrm{d}y \mathrm{d}\sigma = \int_0^\tau u\big|_0^1 \mathrm{d}\sigma = 0. \tag{2.17}$$

从而 $\int_I \frac{1}{\rho}\mathrm{d}y = \int_I \frac{1}{\rho_0}\mathrm{d}y$ 。

通过(2.16)式得到：

$$-\int_I \eta u_y \mathrm{d}y \le \frac{2\mu+\lambda}{2}\int_I \rho u_y^2 \mathrm{d}y + C\sup_{x\in I}\left(\eta-\eta_s\right)^2 \int_I \frac{1}{\rho}\mathrm{d}y + C$$
$$\le \frac{2\mu+\lambda}{2}\int_I \rho u_y^2 \mathrm{d}y + C\sup_{x\in I}\left(\eta-\eta_s\right)^2 + C. \tag{2.18}$$

整理(2.13)与(2.18)式得出：

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\int_I \left[\frac{|u|^2}{2} + \frac{G_1(\rho)}{\rho} + \frac{G_1(\eta)}{\rho} + \frac{\eta\ln\eta}{\rho} + \frac{2\eta\Phi}{\rho}\right]\mathrm{d}y + \frac{2\mu+\lambda}{2}\int_I \rho u_y^2 \mathrm{d}y$$
$$+2\int_I \rho\left|\frac{\eta_y}{\sqrt{\eta}} + \sqrt{\eta}\Phi_y\right|^2 \mathrm{d}y \le C\sup_{x\in I}\left(\eta-\eta_s\right)^2 + C. \tag{2.19}$$

(2.19)两边关于时间方向在 $[0,\tau]$ 上积分，并应用(1.10)，可得，对任意的 $\tau\in[0,T]$，有

$$\int_I \left[\frac{|u|^2}{2} + \frac{G_1(\rho)}{\rho} + \frac{G_1(\eta)}{\rho} + \frac{\eta\ln\eta}{\rho} + \frac{2\eta\Phi}{\rho}\right](\cdot,\tau)\mathrm{d}y$$
$$+\frac{2\mu+\lambda}{2}\int_0^\tau \int_I \rho u_y^2 \mathrm{d}y\mathrm{d}\sigma + 2\int_0^\tau \int_I \rho\left|\frac{\eta_y}{\sqrt{\eta}} + \sqrt{\eta}\Phi_y\right|^2 \mathrm{d}y\mathrm{d}\sigma$$
$$\le \int_I \left[\frac{|u_0|^2}{2} + \frac{G_1(\rho_0)}{\rho_0} + \frac{G_1(\eta_0)}{\rho_0} + \frac{\eta_0\ln\eta_0}{\rho_0} + \frac{2\eta_0\Phi}{\rho_0}\right]\mathrm{d}y + C\int_0^\tau \sup_{x\in I}\left(\eta-\eta_s\right)^2 \mathrm{d}\sigma + C$$
$$\le C\left(E_0,T\right). \tag{2.20}$$

定理 1 证毕。 □

基金项目

国家自然科学基金面上项目(11971357)；广东省自然科学基金面上项目(2023A1515010213)。

参考文献

[1] Ballew, J. (2014) Mathematical Topics in Fluid-Particle Interaction. Ph.D. Thesis, University of Maryland.

[2] Ballew, J. and Trivisa, K. (2012) Suitable Weak Solutions and Low Stratification Singular Limit for a Fluid Particle Interaction Model. *Quarterly of Applied Mathematics*, **70**, 469-494. https://doi.org/10.1090/s0033-569x-2012-01310-2

[3] Ding, S., Huang, B. and Wen, H. (2017) Global Well-Posedness of Classical Solutions to a Fluid-Particle Interaction Model in R^3. *Journal of Differential Equations*, **263**, 8666-8717. https://doi.org/10.1016/j.jde.2017.08.048

[4] Huang, B., Ding, S. and Wen, H. (2016) Local Classical Solutions of Compressible Navier-Stokes-Smoluchowski Equations with Vacuum. *Discrete and Continuous Dynamical Systems—Series S*, **9**, 1717-1752. https://doi.org/10.3934/dcdss.2016072

[5] Huang, B., Ding, S. and Wu, R. (2022) Classical Solutions of the 3D Compressible Fluid-Particle System with a Magnetic Field. *Acta Mathematica Scientia*, **42**, 1585-1606. https://doi.org/10.1007/s10473-022-0417-0

[6] Fang, D., Zi, R. and Zhang, T. (2012) Global Classical Large Solutions to a 1D Fluid-Particle Interaction Model: The Bubbling Regime. *Journal of Mathematical Physics*, **53**, Article ID: 033706. https://doi.org/10.1063/1.3693979

[7] Okada, M. (1989) Free Boundary Value Problems for the Equation of One-Dimensional Motion of Viscous Gas. *Japan Journal of Applied Mathematics*, **6**, 161-177. https://doi.org/10.1007/bf03167921

[8] Okada, M. and Makino, T. (1993) Free Boundary Problem for the Equation of Spherically Symmetric Motion of Viscous Gas. *Japan Journal of Industrial and Applied Mathematics*, **10**, 219-235. https://doi.org/10.1007/bf03167573

[9] Ding, S., Huang, J. and Xia, F. (2013) A Free Boundary Problem for Compressible Hydrodynamic Flow of Liquid Crystals in One Dimension. *Journal of Differential Equations*, **255**, 3848-3879. https://doi.org/10.1016/j.jde.2013.07.039

[10] Huang, J. and Ding, S. (2015) Compressible Hydrodynamic Flow of Nematic Liquid Crystals with Vacuum. *Journal of Differential Equations*, **258**, 1653-1684. https://doi.org/10.1016/j.jde.2014.11.008