

具有相依结构的对偶风险模型的破产概率分析

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摘 要

本文研究了具有相依结构的对偶风险模型, 收益金额的分布规则如下: 在第 i 次的收益参考变量大于第 i 次的收益间隔时间的情形下, 下一次的收益额服从第一类概率分布; 反之, 服从第二类概率分布; 并计算了收益间隔时间与收益金额均服从指数分布时的情形, 通过求导运算推导得到破产概率的积微分方程, 进一步转化为特征方程, 通过对比系数法得到了破产概率的显式表达式; 最后, 为验证模型有效性, 选取参数进行了数值模拟。

关键词

相依结构, 对偶模型, 积 - 微分方程

Analysis of Ruin Probability for the Dual Risk Model with Dependent Structure

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Abstract

This paper studies the dual risk model with dependent structure, and the distribution rule of the gain amount is as follows: when the i gain reference variable is greater than the i gain inter-arrival time, the next gain amount follows the first-type probability distribution; otherwise, it follows the second-type probability distribution. We calculate the case where both the gain inter-arrival time and the gain amount obey the exponential distribution, derive the integro-differential equation for the ruin probability through differentiation, further transform it into a characteristic equation, and obtain the explicit expression of the ruin probability via the coefficient comparison method. Finally, to verify the validity of the model, we select parameters and carry out numerical simulations.

Keywords

Dependent Structure, Dual Model, Integro-Differential Equation

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1. 引言

在保险精算和金融风险文献中,随着风险场景的复杂化,单一收益类型的模型难以满足实际需求,学者们对多态模型的探索展开了广泛的研究:周绍伟等(2007)构建了多险种泊松风险模型,针对索赔额服从指数分布的情形,推导了破产概率的微积分方程及初始资本为 0 时的破产概率表达式,开创了多类型风险下指数分布应用的先河[1];近年来,学者们就双态收益的盈余过程模型展开了广泛的研究,参见[2]-[11];Xie 等(2018)拓展了马尔可夫调制框架,聚焦双态理赔过程的相依结构,利用 Copula 函数刻画风险关联性,提升了模型的现实贴合度[12]。

尽管现有研究已实现从单一风险到多态风险、从固定参数到随机调制的拓展,但仍存在显著不足:多态风险研究多聚焦于理赔过程的类型划分,针对双态收益结构的盈余过程探讨较为有限;Zou (2014)等学者虽简化了指数分布下的破产概率求解,但未涉及双态收益场景下的积分项处理,难以直接迁移至复杂收益结构[13]。本文以 Cramér-Lundberg 经典模型为研究基础,引入双态收益结构:根据前一次收益金额是否超过阈值,下一次收益额分别服从不同参数的指数分布,完善了多态收益盈余过程的理论体系。

本文拓展了盈余风险模型的收益结构,丰富了收益形态类型,通过严谨的数学推导,得到双态收益与盈余过程的积微分方程及破产概率解析求解,通过求导转换、积分项简化与特征方程求解等方法,完善了双态收益场景下的风险量化评估体系,验证了指数分布假设下双态收益盈余模型的破产概率解析可解性。

2. 模型介绍

考虑以下对偶模型,盈余过程定义为

$$U(t) = u - ct + \sum_{i=1}^{N(t)} Y_i$$

其中 $u \geq 0$ 表示初始盈余, $c > 0$ 为支出率,收益序列 $\{Y_i\}_{i=1}^{\infty}$ 是独立同分布的随机变量,其概率密度函数为 f_Y , 累计分布函数为 F_Y , $\sum_{i=1}^{N(t)} Y_i$ 表示到达时间 t 前的累积获利; $N(t) = \max\{k : T_0 + T_1 + \dots + T_k \leq t, T_0 = 0\}$ 为时间 t 前的收益次数,是 poisson 过程,参数为 λ , 其中 $\{T_i, i \geq 1\}$ 表示第 $i-1$ 次与第 i 次收益的间隔时间, $\{T_i, i \geq 1\}$ 服从参数为 λ 的指数分布,且是独立同分布 (i.i.d.) 的随机变量序列;

设 $\{M_i, i \geq 1\}$ 为影响收益质量的外部状态变量,其与收益间隔时间 Y_i 的比较决定了未来收益的分布模式, $\{M_i, i \geq 1\}$ 为独立同分布的非负随机变量,分布函数是 F_M , 我们假设收益金额的分布规则如下:若 $M_i > T_i$, 则下一次收益额服从 $B_1(\cdot)$, 概率密度为 $b_1(\cdot)$; 反之,服从 $B_2(\cdot)$ 。这里 $\{M_i, i \geq 1\}$ 与 $\{T_i, i \geq 1\}$ 及 $\{B_i, i = 1, 2\}$ 是相互独立的。

在一般情况下,破产概率的微积分方程可写为:

$$\begin{aligned}\phi(u) = & e^{-\frac{\lambda u}{c}} + \lambda \int_0^{\frac{u}{c}} e^{-\lambda t} P(M > t) \int_0^{\infty} \phi(u - ct + x) B_1(dx) dt \\ & + \lambda \int_0^{\frac{u}{c}} e^{-\lambda t} P(M \leq t) \int_0^{\infty} \phi(u - ct + x) B_2(dx) dt\end{aligned}$$

令 $s = u - ct$ ，得到：

$$\begin{aligned}\phi(u) = & e^{-\frac{\lambda u}{c}} + \frac{\lambda}{c} \int_0^u e^{-\lambda \left(\frac{u-s}{c}\right)} P\left(M > \frac{u-s}{c}\right) \int_0^{\infty} \phi(s+x) B_1(dx) ds \\ & + \frac{\lambda}{c} \int_0^u e^{-\lambda \left(\frac{u-s}{c}\right)} P\left(M \leq \frac{u-s}{c}\right) \int_0^{\infty} \phi(s+x) B_2(dx) ds\end{aligned}$$

基于以上条件，得到安全负载条件为：

$$\frac{c}{\lambda} \leq E(B_1)P(M_i > T_i) + E(B_2)P(M_i \leq T_i)$$

3. 指数分布下的破产概率

由于上式是积分方程，直接求解很困难，故假设 M, B_1, B_2 服从参数分别为 δ_M, α, β 的指数分布，则上式可重新写为：

$$\begin{aligned}\phi(u) = & e^{-\frac{\lambda u}{c}} + \frac{\lambda}{c} \int_0^u e^{-\lambda \left(\frac{u-s}{c}\right)} e^{-\delta_M \left(\frac{u-s}{c}\right)} \int_0^{\infty} \phi(s+x) \alpha e^{-\alpha x} dx ds \\ & + \frac{\lambda}{c} \int_0^u e^{-\lambda \left(\frac{u-s}{c}\right)} \left(1 - e^{-\delta_M \left(\frac{u-s}{c}\right)}\right) \int_0^{\infty} \phi(s+x) \beta e^{-\beta x} dx ds\end{aligned} \quad (3.1)$$

要得到关于 u 的积 - 微分方程，要对 $\phi(u)$ 求导，得到：

$$\begin{aligned}\phi'(u) = & \left(-\frac{\lambda}{c}\right) e^{-\frac{\lambda u}{c}} + \frac{\lambda \alpha}{c} \int_0^{\infty} \phi(u+x) e^{-\alpha x} dx \\ & - \frac{\lambda(\lambda + \delta_M)}{c^2} \int_0^u e^{-(\lambda + \delta_M) \left(\frac{u-s}{c}\right)} \left(\int_0^{\infty} \phi(s+x) \alpha e^{-\alpha x} dx\right) ds \\ & - \frac{\lambda}{c^2} \int_0^u e^{-\lambda \left(\frac{u-s}{c}\right)} \left[\lambda \left(1 - e^{-\delta_M \left(\frac{u-s}{c}\right)}\right) + \delta_M e^{-\delta_M \left(\frac{u-s}{c}\right)}\right] \left(\int_0^{\infty} \phi(s+x) \beta e^{-\beta x} dx\right) ds\end{aligned} \quad (3.2)$$

在等式(3.1)的两端乘以 $\frac{\lambda + \delta_M}{c}$ 再加式(3.2)，得到：

$$\left(\frac{\lambda + \delta_M}{c} I + \wp\right) \phi(u) = \frac{\delta_M}{c} e^{-\frac{\lambda u}{c}} + \frac{\lambda}{c} \sigma_1(u) + \frac{\lambda \delta_M}{c^2} \int_0^u e^{-\lambda \left(\frac{u-s}{c}\right)} \sigma_2(s) ds \quad (3.3)$$

我们定义

$$\left(\frac{\lambda + \delta_M}{c} I + \wp\right) \phi(u) = g(u) \quad (3.4)$$

对 u 求导，得到：

$$g'(u) = -\frac{\lambda\delta_M}{c^2}e^{-\frac{\lambda u}{c}} + \frac{\lambda}{c}\sigma_1'(u) + \frac{\lambda\delta_M}{c^2}\left[\sigma_2(u) - \frac{\lambda}{c}\int_0^u e^{-\lambda\left(\frac{u-s}{c}\right)}\sigma_2(s)ds\right] \quad (3.5)$$

在等式(3.4)的两端乘以 $\frac{\lambda}{c}$ 再加(3.5)式, 得到

$$\left(\frac{\lambda+\delta_M}{c}I + \wp\right)\left(\frac{\lambda}{c}I + \wp\right)\phi(u) = \frac{\lambda}{c}\left(\frac{\lambda}{c}I + \wp\right)\sigma_1(u) + \frac{\lambda\delta_M}{c^2}\sigma_2(u) \quad (3.6)$$

其中,

$$\sigma_1(u) = \int_0^\infty \phi(u+x)\alpha e^{-\alpha x}dx, \sigma_2(u) = \int_0^\infty \phi(u+x)\beta e^{-\beta x}dx$$

上式可写为:

$$(\wp - \alpha I)(\wp - \beta I)\left(\frac{\lambda+\delta_M}{c}I + \wp\right)\left(\frac{\lambda}{c}I + \wp\right)\phi(u) = 0$$

即:

$$\left\{\wp^4 + \left(\frac{2\lambda+\delta_M}{c} - \alpha - \beta\right)\wp^3 + \left[\frac{\lambda(\lambda+\delta_M)}{c^2} - \frac{(\alpha+\beta)(2\lambda+\delta_M)}{c} + \alpha\beta\right]\wp^2 + \left[\frac{\alpha\beta(2\lambda+\delta_M)}{c} - \frac{\lambda(\alpha+\beta)(\lambda+\delta_M)}{c^2}\right]\wp + \frac{\lambda\alpha\beta(\lambda+\delta_M)}{c^2}\right\}\phi(u) = 0$$

进而得到特征方程为:

$$\left\{z^4 + \left(\frac{2\lambda+\delta_M}{c} - \alpha - \beta\right)z^3 + \left[\frac{\lambda(\lambda+\delta_M)}{c^2} - \frac{(\alpha+\beta)(2\lambda+\delta_M)}{c} + \alpha\beta\right]z^2 + \left[\frac{\alpha\beta(2\lambda+\delta_M)}{c} - \frac{\lambda(\alpha+\beta)(\lambda+\delta_M)}{c^2}\right]z + \frac{\lambda\alpha\beta(\lambda+\delta_M)}{c^2}\right\} = 0 \quad (3.7)$$

下面对(3.7)进一步分析其特征根的分布性质, 以确保破产概率解的有效性, 为讨论根的分布情况, 引入辅助函数:

$$f(r) = \left\{z^4 + \left(\frac{2\lambda+\delta_M}{c} - \alpha - \beta\right)z^3 + \left[\frac{\lambda(\lambda+\delta_M)}{c^2} - \frac{(\alpha+\beta)(2\lambda+\delta_M)}{c} + \alpha\beta\right]z^2 + \left[\frac{\alpha\beta(2\lambda+\delta_M)}{c} - \frac{\lambda(\alpha+\beta)(\lambda+\delta_M)}{c^2}\right]z\right\}$$

$$g(r) = \frac{\lambda\alpha\beta(\lambda+\delta_M)}{c^2}$$

选取充分大的 $R > 0$, 在区域 $|r| \leq R$ 内, 在边界上可以验证 $|f(r)| > |g(r)|$, 则由 Rouché 定理可知, $f(r)$ 与 $f(r)+g(r)$ 在该区域内具有相同数量的零点, 进一步分析可以证明, (3.7)式子在右半平面至多有两个实根, 且有两个负实根, 这保证了破产概率在 $u \rightarrow \infty$ 时趋于零, 进一步验证了该风险模型的合理性。

由 Zou (2014), 令:

$$\phi(u) = c_1 e^{\tau_1 u} + c_2 e^{\tau_2 u} + c_3 e^{\tau_3 u} + c_4 e^{\tau_4 u}$$

代入到(3.1)式, 得:

$$\begin{aligned}
& c_1 e^{z_1 u} + c_2 e^{z_2 u} + c_3 e^{z_3 u} + c_4 e^{z_4 u} \\
&= e^{-\frac{\lambda u}{c}} + \frac{\lambda}{c} \int_0^u e^{-\left(\frac{\lambda+s}{c}\right)\left(\frac{u-s}{c}\right)} \int_0^\infty [c_1 e^{z_1(s+x)} + c_2 e^{z_2(s+x)} + c_3 e^{z_3(s+x)} + c_4 e^{z_4(s+x)}] \alpha e^{-\alpha x} dx ds \\
&\quad + \frac{\lambda}{c} \int_0^u e^{-\left(\frac{\lambda+s}{c}\right)\left(\frac{u-s}{c}\right)} \left(1 - e^{-\delta_M \left(\frac{u-s}{c}\right)}\right) \int_0^\infty [c_1 e^{z_1(s+x)} + c_2 e^{z_2(s+x)} + c_3 e^{z_3(s+x)} + c_4 e^{z_4(s+x)}] \beta e^{-\beta x} dx ds
\end{aligned}$$

极端积分项可以得到:

$$\begin{aligned}
& c_1 e^{z_1 u} \left(1 - \frac{\lambda \Delta_1}{e(\lambda + \delta_M - z_1 c)} - \frac{c_1 \Lambda_1}{e(\lambda - cz_1)} + \frac{c_1 \lambda \Lambda_1}{\lambda + \delta_M - z_1 c} \frac{1}{e}\right) \\
&+ c_2 e^{z_2 u} \left(1 - \frac{\lambda \Delta_2}{e(\lambda + \delta_M - z_2 c)} - \frac{c_2 \Lambda_2}{e(\lambda - cz_2)} + \frac{c_2 \lambda \Lambda_2}{\lambda + \delta_M - z_2 c} \frac{1}{e}\right) \\
&+ c_3 e^{z_3 u} \left(1 - \frac{\lambda \Delta_3}{e(\lambda + \delta_M - z_3 c)} - \frac{c_3 \Lambda_3}{e(\lambda - cz_3)} + \frac{c_3 \lambda \Lambda_3}{\lambda + \delta_M - z_3 c} \frac{1}{e}\right) \\
&+ c_4 e^{z_4 u} \left(1 - \frac{\lambda \Delta_4}{e(\lambda + \delta_M - z_4 c)} - \frac{c_4 \Lambda_4}{e(\lambda - cz_4)} + \frac{c_4 \lambda \Lambda_4}{\lambda + \delta_M - z_4 c} \frac{1}{e}\right) \\
&+ c_1 \lambda \Delta_1 \frac{1}{\lambda + \delta_M - z_1 c} e^{-\left(\frac{\lambda + \delta_M}{c}\right)u} - c_1 \lambda \Lambda_1 \frac{1}{\lambda + \delta_M - z_1 c} e^{-\left(\frac{\lambda + \delta_M}{c}\right)u} \\
&+ c_2 \lambda \Delta_2 \frac{1}{\lambda + \delta_M - z_2 c} e^{-\left(\frac{\lambda + \delta_M}{c}\right)u} - c_2 \lambda \Lambda_2 \frac{1}{\lambda + \delta_M - z_2 c} e^{-\left(\frac{\lambda + \delta_M}{c}\right)u} \\
&+ c_3 \lambda \Delta_3 \frac{1}{\lambda + \delta_M - z_3 c} e^{-\left(\frac{\lambda + \delta_M}{c}\right)u} - c_3 \lambda \Lambda_3 \frac{1}{\lambda + \delta_M - z_3 c} e^{-\left(\frac{\lambda + \delta_M}{c}\right)u} \\
&+ c_4 \lambda \Delta_4 \frac{1}{\lambda + \delta_M - z_4 c} e^{-\left(\frac{\lambda + \delta_M}{c}\right)u} - c_4 \lambda \Lambda_4 \frac{1}{\lambda + \delta_M - z_4 c} e^{-\left(\frac{\lambda + \delta_M}{c}\right)u} \\
&+ c_1 \lambda \Lambda_1 \frac{1}{\lambda - cz_1} e^{-\frac{\lambda}{c}u} + c_2 \lambda \Lambda_2 \frac{1}{\lambda - cz_2} e^{-\frac{\lambda}{c}u} + c_3 \lambda \Lambda_3 \frac{1}{\lambda - cz_3} e^{-\frac{\lambda}{c}u} \\
&+ c_4 \lambda \Lambda_4 \frac{1}{\lambda - cz_4} e^{-\frac{\lambda}{c}u} - e^{-\frac{\lambda}{c}u} = 0
\end{aligned}$$

$$\text{其中, } \Delta_i = \frac{\alpha}{z_i + \alpha}, \Lambda_i = \frac{\beta}{z_i + \beta}, i=1,2,3,4$$

从而由比较系数法得到:

$$\begin{cases} \sum_{i=1}^4 c_i \lambda \Delta_i \frac{1}{\lambda + \delta_M - z_i c} = 0 \\ \sum_{i=1}^4 c_i \lambda \Lambda_i \frac{1}{\lambda + \delta_M - z_i c} = 0 \\ \sum_{i=1}^4 c_i \lambda \Lambda_i \frac{1}{\lambda - cz_i} = 1 \\ c_1 + c_2 + c_3 + c_4 = 1 \end{cases}$$

设该方程组的系数矩阵为:

$$M = \begin{vmatrix} \frac{\lambda\Delta_1}{\lambda + \delta_M - z_1c} & \frac{\lambda\Delta_2}{\lambda + \delta_M - z_2c} & \frac{\lambda\Delta_3}{\lambda + \delta_M - z_3c} & \frac{\lambda\Delta_4}{\lambda + \delta_M - z_4c} \\ \frac{\lambda\Lambda_1}{\lambda + \delta_M - z_1c} & \frac{\lambda\Lambda_2}{\lambda + \delta_M - z_2c} & \frac{\lambda\Lambda_3}{\lambda + \delta_M - z_3c} & \frac{\lambda\Lambda_4}{\lambda + \delta_M - z_4c} \\ \frac{\lambda\Lambda_1}{\lambda - cz_1} & \frac{\lambda\Lambda_2}{\lambda - cz_2} & \frac{\lambda\Lambda_3}{\lambda - cz_3} & \frac{\lambda\Lambda_4}{\lambda - cz_4} \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

由克拉默法则，可解得：

$$c_1 = \frac{M_1}{M}, c_2 = \frac{M_2}{M}, c_3 = \frac{M_3}{M}, c_4 = \frac{M_4}{M}$$

其中：

$$M_1 = \begin{vmatrix} 0 & \frac{\lambda\Delta_2}{\lambda + \delta_M - z_2c} & \frac{\lambda\Delta_3}{\lambda + \delta_M - z_3c} & \frac{\lambda\Delta_4}{\lambda + \delta_M - z_4c} \\ 0 & \frac{\lambda\Lambda_2}{\lambda + \delta_M - z_2c} & \frac{\lambda\Lambda_3}{\lambda + \delta_M - z_3c} & \frac{\lambda\Lambda_4}{\lambda + \delta_M - z_4c} \\ 1 & \frac{\lambda\Lambda_2}{\lambda - cz_2} & \frac{\lambda\Lambda_3}{\lambda - cz_3} & \frac{\lambda\Lambda_4}{\lambda - cz_4} \\ 1 & 1 & 1 & 1 \end{vmatrix} \quad M_2 = \begin{vmatrix} \frac{\lambda\Delta_1}{\lambda + \delta_M - z_1c} & 0 & \frac{\lambda\Delta_3}{\lambda + \delta_M - z_3c} & \frac{\lambda\Delta_4}{\lambda + \delta_M - z_4c} \\ \frac{\lambda\Lambda_1}{\lambda + \delta_M - z_1c} & 0 & \frac{\lambda\Lambda_3}{\lambda + \delta_M - z_3c} & \frac{\lambda\Lambda_4}{\lambda + \delta_M - z_4c} \\ \frac{\lambda\Lambda_1}{\lambda - cz_1} & 1 & \frac{\lambda\Lambda_3}{\lambda - cz_3} & \frac{\lambda\Lambda_4}{\lambda - cz_4} \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

$$M_3 = \begin{vmatrix} \frac{\lambda\Delta_1}{\lambda + \delta_M - z_1c} & \frac{\lambda\Delta_2}{\lambda + \delta_M - z_2c} & 0 & \frac{\lambda\Delta_4}{\lambda + \delta_M - z_4c} \\ \frac{\lambda\Lambda_1}{\lambda + \delta_M - z_1c} & \frac{\lambda\Lambda_2}{\lambda + \delta_M - z_2c} & 0 & \frac{\lambda\Lambda_4}{\lambda + \delta_M - z_4c} \\ \frac{\lambda\Lambda_1}{\lambda - cz_1} & \frac{\lambda\Lambda_2}{\lambda - cz_2} & 1 & \frac{\lambda\Lambda_4}{\lambda - cz_4} \\ 1 & 1 & 1 & 1 \end{vmatrix} \quad M_4 = \begin{vmatrix} \frac{\lambda\Delta_1}{\lambda + \delta_M - z_1c} & \frac{\lambda\Delta_2}{\lambda + \delta_M - z_2c} & \frac{\lambda\Delta_3}{\lambda + \delta_M - z_3c} & 0 \\ \frac{\lambda\Lambda_1}{\lambda + \delta_M - z_1c} & \frac{\lambda\Lambda_2}{\lambda + \delta_M - z_2c} & \frac{\lambda\Lambda_3}{\lambda + \delta_M - z_3c} & 0 \\ \frac{\lambda\Lambda_1}{\lambda - cz_1} & \frac{\lambda\Lambda_2}{\lambda - cz_2} & \frac{\lambda\Lambda_3}{\lambda - cz_3} & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

4. 数值模拟

选取参数 $\lambda = 1.5$ ， $c = 0.8$ ， $\alpha = 2$ ， $\beta = 1.5$ ， $\delta_M = 0.15$ ，该参数选取满足安全负载条件，进而得到特征方程为：

$$z^4 - 0.3125z^3 - 5.625z^2 + 0.703125z + 7.59375 = 0$$

解得：

$$z_1 \approx -2.224, z_2 \approx -0.669, z_3 \approx 0.754, z_4 \approx 1.389$$

代入，可得：

$$\begin{cases} \sum_{i=1}^4 1.5c_i \Delta_i \frac{1}{1.65 - 0.8z_i} = 0 \\ \sum_{i=1}^4 1.5c_i \Lambda_i \frac{1}{1.65 - 0.8z_i} = 0 \\ \sum_{i=1}^4 1.5c_i \Lambda_i \frac{1}{1.5 - 0.8z_i} = 1 \\ \sum_{i=1}^4 c_i = 1 \end{cases}$$

从而可解出：

$$c_1 = 0.482, c_2 = 0.366, c_3 = 0.148, c_4 = 0.004$$

故破产概率的表达式可写为：

$$\phi(u) = 0.482e^{-2.224u} + 0.366e^{-0.669u} + 0.148e^{0.754u} + 0.004e^{1.389u}$$

基于以上结果，绘制图像：

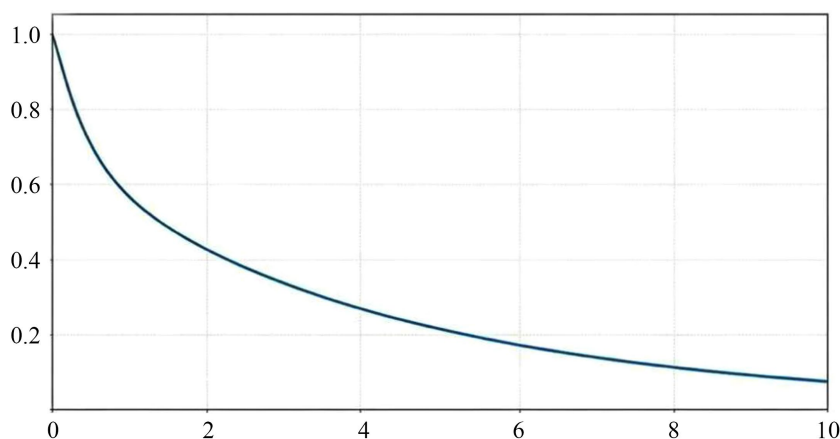


Figure 1. Plot of the ruin probability varying with surplus

图 1. 破产概率随盈余改变变化图像

图 1 表示随着初始盈余从 0 逐步增加至 60，破产概率从 1 持续下降，最终趋近于 0；例如盈余为 10 时，破产概率降至 0.6 左右；盈余为 30 时，概率降至 0.3 左右，该趋势完全符合风险理论，即初始盈余越多，企业抵御固定支出压力和收益波动的能力越强，破产可能性越低，说明模型能准确刻画该相依对偶风险模型下的盈余风险逻辑，这对企业具有直接应用价值：可根据目标破产概率，反推所需的最小初始盈余；直观展示“增加初始资本”对降低破产风险的边际效益。

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