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均值-方差准则下马尔科夫调制的最优投资再保险策略

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摘 要

本文研究了马尔科夫调制下基于均值-方差准则的最优投资再保险策略问题。容许保险公司购买比例再保险，同时投资一个无风险资产和两个相依的风险资产，风险资产的相依性由价格过程受到的共同冲击来刻画。应用随机线性二次型控制理论求出最优投资再保险策略，并通过拉格朗日对偶定理得到有效前沿。

关键词

均值-方差，马尔科夫调制，相依风险资产，随机线性二次型控制，有效前沿

Optimal Mean-Variance Investment Reinsurance Strategy under Markov Modulated Model

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Abstract

Based on the mean variance criterion, this paper studies the optimal investment reinsurance strategy under Markov modulated model, and assumes that an insurer is allowed to purchase proportional reinsurance business and invest a risk-free asset and two dependent risky assets in the financial market. And the dependence of risky assets is characterized by a common shock of the price process. Stochastic linear quadratic control theory is used to find the optimal investment reinsurance strategy, and the effective frontier is obtained by Lagrange duality theorem.

Keywords

Mean-Variance, Markov-Modulated, Dependent Risky Assets, Stochastic Linear Quadratic Control, Efficient Frontier

1. 介绍

Markowitz (1952) [1]提出的均值-方差投资组合理论在金融工程中一直发挥着重要的作用. Bäuerle (2005) [2]首次将均值-方差准则应用于保险精算, 并得到了有无约束两种情形下最优再保险策略的解, 引起了许多学者的广泛关注. Bai 和Zhang (2008) [3]研究了不同风险模型在均值方差准则下的最优再保险投资策略问题, 通过求解HJB 方程得到最优策略的解析表达. Yan 和Li (2011) [4]研究了基于均值-方差准则, 保险人分别仅购买再保险和仅投资两种情况下的最优策略问题. Bi 等(2019) [5] 研究了均值-方差准则下索赔相依的最优投资再保险策略问题. 其他相关研究可见文献 [6–8]等.

Hamilton (1989) [9]首次在金融工程中引入了带有马尔科夫调制的动态模型, 依据美国GNP数据进行了实证研究. 近年来, 大量文章开始将马尔科夫调制模型应用到最优投资再保险策略问题中. Zhou 等(2003) [10] 研究了在马尔科夫调制下, 风险资产由n支股票组成的最优投资组合问题. Ping 和Yam (2013) [11]研究了马尔科夫调制下的最优投资再保险策略问题; Bi 等(2019) [5]研究在马尔科夫调制下两个相依索赔过程的最优投资再保险问题, 利用随机线性二次型控制技术, 得到了最优

投资再保险策略和有效前沿的解析表达式; 张彩斌等(2021) [12]研究了在马尔科夫调制下, 当盈余跳过程与风险资产跳过程相依时的最优投资再保险问题, 利用随机控制理论和广义HJB 方程得到问题的最优解. 在实际中, 一些重要市场参数如利率、收益率、波动率等会随整体经济状态的变化而改变, 马尔科夫调制模型很好地刻画了这种现象. 本文中将用马尔科夫调制的跳扩散模型来刻画风险资产的价格过程.

目前, 大多数文献在研究再保险投资策略的相依性问题时, 多是考虑险种之间存在相依关系. 事实上, 保险公司进行投资行为时, 风险资产之间也会存在相依性, 但关于此类问题的研究却鲜见文献. 投资过程中, 风险资产的价格不仅受到自身经营(或称非系统性风险)的影响, 通常用布朗运动来刻画, 同时也会受到外界环境(或称系统性风险)的影响, 通常用跳过程来刻画. 我们假设金融市场突然受到罕见的、意外的外界冲击，如疫情的突然爆发等, 风险资产价格出现剧烈变化. 此时不同风险资产受到共同冲击而存在相依性，我们用跳过程相依来刻画. 本文研究的重点是, 在马尔科夫调制下, 保险公司购买比例再保险, 同时投资一个无风险资产和两个相依的风险资产, 基于均值方差准则, 应用随机线性二次型控制理论求出最优投资再保险策略解析表达, 并通过拉格朗日对偶定理得到有效前沿.

2. 模型描述

给定一个完备的概率空间$(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$, σ- 流$\{\mathcal{F}_t\}_{t\geq 0}$满足通常条件. $\{W_n(t), t \geq 0, n = 0, 1, 2\}$ 是$(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ 上的标准布朗运动, $\{\alpha(t), t \geq 0\}$定义在$(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$, 取值于$\mathcal{N} = \{1, 2, \cdot\cdot\cdot, d\}$上连续时间有限状态的齐次马尔科夫链, d 取正整数, 其转移概率为

$$p_{ij}(t) = P(\alpha(t) = j \mid \alpha(0) = i), \quad t \geq 0, i, j = 1, 2, \ldots, d.$$

记$Q = (q_{ij})_{d\times d}$为马氏链$\alpha(t)$的强度矩阵

$$q_{ij} = \lim_{t\to 0^+} \frac{p_{ij}(t)}{t}, i \neq j, \quad q_{ii} = \lim_{t\to 0^+} \frac{p_{ii}(t) - 1}{t}, i = 1, 2, \cdots, d,$$

对每个i 有$\sum_{j\in\mathcal{N}} q_{ij} = 0, q_{ij} > 0 (i \neq j)$, 且$W_n(t)$ 和$\alpha(t)$ 相互独立.

记$C(t)$是保险公司索赔额, 服从如下扩散过程

$$dC(t) = a(t, \alpha(t))dt - b(t, \alpha(t))dW_0(t), \tag{2.1}$$

其中$a(t, i) > 0$和$b(t, i) > 0$, $i \in \mathcal{N}$. 由此可得盈余过程$R(t)$有如下表述

$$dR(t) = c(t, \alpha(t))dt - dC(t), \tag{2.2}$$

其中$c(t, i)$是保费率, $i \in \mathcal{N}$.

一般地, 保险公司通过购买再保险业务来转移风险, 本文假定保险公司按比例再保险缴纳保费. $c_1(t, i)$表示保险公司的保费率, $c_2(t, i)$表示再保险公司保费率, $\theta(t, i)$, $\eta(t, i)$ 分别为保险公司和再保险公司的安全系数, $\eta(t, i) > \theta(t, i)$, $i \in \mathcal{N}$. 令$\xi(t)$表示再保险公司在t时刻索赔的比例, $\xi(t) \leq 1$. 基

于期望保费原理则有$c_1(t,i)=a(t,i)(1+\theta(t,i))$, $c_2(t,i)=a(t,i)\xi(t)(1+\eta(t,i))$. 此时保险公司盈余过程

$$\begin{aligned} dR(t)=&c_1(t,\alpha(t))\mathrm{d}t-(1-\xi(t))dC(t)-c_2(t,\alpha(t))dt \\ =&a(t,\alpha(t))(\theta(t,\alpha(t))-\eta(t,\alpha(t))\xi(t))dt+b(t,\alpha(t))(1-\xi(t))dW_0(t). \end{aligned} \tag{2.3}$$

保险公司往往追求利润的最大化, 在金融市场存在投资行为. 假定金融市场由一个无风险债券和两个相依的风险资产组成. 保险公司在t 时刻投资于无风险资产的价格过程$B(t)$ 满足

$$\begin{cases} dB(t)=r_0(t,\alpha(t))B(t)dt, \quad t\in[0,T], \\ B(0)=B_0>0, \end{cases} \tag{2.4}$$

其中$r_0(t,i)>0$是无风险债券的利率, $i=\mathcal{N}$.

在t 时刻风险资产的价格过程$S_1(t)$，$S_2(t)$分别满足

$$\begin{cases} dS_1(t)=S_1(t)\left[r_1(t,\alpha(t))dt+b_1(t,\alpha(t))dW_1(t)+dY(t)\right], t\in[0,T], \\ S_1(0)=s_1>0, \end{cases} \tag{2.5}$$

$$\begin{cases} dS_2(t)=S_2(t)\left[r_2(t,\alpha(t))dt+b_2(t,\alpha(t))dW_2(t)+dM(t)\right], t\in[0,T], \\ S_2(0)=s_2>0, \end{cases} \tag{2.6}$$

其中$r_1(t,i)(\geq r_0(t,i))$, $r_2(t,i)(\geq r_0(t,i))$是保险公司投资于风险市场的收益率, $b_1(t,i)(>0), b_2(t,i)(>0)$是风险市场的的波动率系数. 假定

$$Y(t)=\sum_{k=1}^{N_1(t)+N_0(t)}Y_k, \quad M(t)=\sum_{k=1}^{N_2(t)+N_0(t)}M_k \tag{2.7}$$

是两个复合泊松过程, 交叉计数过程$\{N_0(t)\}_{t\geq0}$刻画风险资产的相依性. 其中$\{N_0(t)\}_{t\geq0}$, $\{N_1(t)\}_{t\geq0}$, $\{N_2(t)\}_{t\geq0}$ 是三个相互独立的泊松过程, 强度系数分别为$\lambda_0(t,i)$, $\lambda_1(t,i)$, $\lambda_2(t,i)$, $i\in\mathcal{N}$. 另外, 假定$\{Y_k,k\geq1\}(\{M_k,k\geq1\})$为独立同分布的随机变量, 有共同分布$F_Y(\cdot)(F_M(\cdot))$, 有$E(Y)=\mu_{11}>0$, $E(Y^2)=\mu_{12}>0$, $E(M)=\mu_{21}>0$, $E(M^2)=\mu_{22}>0$. $\{Y_k,k\geq1\}$和$\{M_k,k\geq1\}$相互独立, 且与$\{N_1(t)\}_{t\geq0}$ 和$\{N_2(t)\}_{t\geq0}$ 也相互独立.

现保险公司考虑将一部分资金投资于金融市场. 记$l_1(t)$，$l_2(t)$为在t 时刻投资于金融市场两风险资产的额度, $X(t)$表示t 时刻的财富过程, 不考虑交易费用等, 则$X(t)$ 有如下表达

$$\begin{aligned} dX(t)=&dR(t)+\frac{l_1(t)}{S_1(t)}dS_1(t)+\frac{l_2(t)}{S_2(t)}dS_2(t)+\frac{X(t)-l_1(t)-l_2(t)}{B(t)}dB(t) \\ =&[X(t)r_0(t,\alpha(t))+a(t,\alpha(t))(\theta(t,\alpha(t))-\eta(t,\alpha(t)\xi(t)) \\ &+l_1(t)(r_1(t,\alpha(t))-r_o(t,\alpha(t)))+l_2(t)(r_2(t,\alpha(t))-r_o(t,\alpha(t)))]dt \\ &+b(t,\alpha(t))(1-\xi(t))dW_0(t)+l_1(t)b_1(t,\alpha(t))dW_1(t)+l_2(t)b_2(t,\alpha(t))dW_2(t) \\ &+l_1(t)dY(t)+l_2(t)dM(t), \end{aligned} \tag{2.8}$$

其中$X(0)=x_0$, $\alpha(0)=i_0$, 且$r_0(t,i)$, $a(t,i)$, $\theta(t,i)$, $\eta(t,i)$, $r_1(t,i)$, $r_2(t,i)$是$t\in[0,T]$上已知连续函数.

为了便于求解, 我们应用Grandell(1991) [13]中方法将(2.7)处理如下

$$dY(t)=(\lambda_0(t,i)+\lambda_1(t,i))\mu_{11}(t,i)dt-\sqrt{(\lambda_0(t,i)+\lambda_1(t,i))\mu_{12}(t,i)}dW_{11}(t),$$

$$dM(t)=(\lambda_0(t,i)+\lambda_2(t,i))\mu_{21}(t,i)dt-\sqrt{(\lambda_0(t,i)+\lambda_2(t,i))\mu_{22}(t,i)}dW_{22}(t),$$

其中标准布朗运动$W_{11}(t)$, $W_{22}(t)$与$\{W_n(t),t\geq 0\}$，$n=0,1,2$独立. 标准布朗运动W_{11}, W_{22} 的相关系数

$$\rho(t,i)=\frac{\lambda_0(t,i)\mu_{11}(t,i)\mu_{21}(t,i)}{\sigma_1(t,i)\sigma_2(t,i)}\in(0,1).$$

令

$$\varepsilon_1(t,i)=(\lambda_0(t,i)+\lambda_1(t,i))\mu_{11}(t,i),\qquad \varepsilon_2(t,i)=(\lambda_0(t,i)+\lambda_2(t,i))\mu_{21}(t,i),$$

$$\sigma_1^2(t,i)=(\lambda_0(t,i)+\lambda_1(t,i))\mu_{12}(t,i),\qquad \sigma_2^2(t,i)=(\lambda_0(t,i)+\lambda_2)\mu_{22}(t,i).$$

则有

$$\begin{aligned}dX(t)&=dR(t)+\frac{l_1(t)}{S_1(t)}dS_1(t)+\frac{l_2(t)}{S_2(t)}dS_2(t)+\frac{X(t)-l_1(t)-l_2(t)}{B(t)}dB(t)\\&=[X(t)r_0(t,\alpha(t))+a(t,\alpha(t))(\theta(t,\alpha(t))-\eta(t,\alpha(t))\xi(t))\\&+l_1(t)(r_1(t,\alpha(t))-r_o(t,\alpha(t))+\varepsilon_1(t,\alpha(t)))\\&+l_2(t)(r_2(t,\alpha(t))-r_o(t,\alpha(t))+\varepsilon_2(t,\alpha(t)))]dt\end{aligned}\tag{2.9}$$

$$+b(t,\alpha(t))(1-\xi(t))dW_0(t)+l_1(t)b_1(t,\alpha(t))dW_1(t)+l_2(t)b_2(t,\alpha(t))dW_2(t)$$

$$-\sqrt{l_1^2(t)\sigma_1^2(t,\alpha(t))+l_2^2(t)\sigma_2^2(t,\alpha(t))+2\rho(t,\alpha(t)))l_1(t)l_2(t)\sigma_1(t,\alpha(t))\sigma_2(t,\alpha(t))}dW(t).$$

定义1 记$\pi(t)=(\xi(t),l_1(t),l_2(t))$表示一个投资再保险策略, 称$\pi(t)$是可容许的. 满足以下条件

（1）$\pi(t)$是$\mathcal{F}$可测的, $t\in[0,T]$.

（2）$0\leq\xi(t)\leq 1, l_1(t)\geq 0, l_2(t)\geq 0$.

（3）$\mathbb{E}[\int_0^t(\xi(s))^2ds]<\infty$, $\mathbb{E}[\int_0^t(l_1(s))^2ds]<\infty$, $\mathbb{E}[\int_0^t(l_2(s))^2ds]<\infty$, $t\in[0,T]$.

3. 最优投资再保险策略

假定保险公司预期在终端时刻T得到的总财富为z, 即$\mathbb{E}[X(T)]=z$. 则需要在此目标下找到可行的最优投资再保险策略, 使得T时刻财富的风险达到最小, 这里我们用T 时刻财富的方差来度量风险, 即

$$\mathrm{Var}\,X(T)=\mathbb{E}\left\{[X(T)-\mathbb{E}X(T)]^2\right\}=\mathbb{E}\left\{[X(T)-z]^2\right\}.\tag{3.1}$$

运用均值方差准则, 上述问题表示为

$$\min \operatorname{Var} X(T)=\mathbb{E}\left\{[X(T)-z]^2\right\},$$
$$\text{使得}\begin{cases}\mathbb{E}X(T)=z,\\ \pi\in\Pi,\\ (X(\cdot),\pi(\cdot))\ \text{满足(2.9)}.\end{cases}\tag{3.2}$$

其中Π 为所有可容许策略$\pi(t)$的集合. 上述问题成为马尔科夫调制的均值方差问题.

下面我们讨论问题(3.2)投资策略$\pi(t)$的容许可行解. 首先定义如下常微分方程组

$$\begin{cases}\dot{\psi}(t,i)=-r(t,i)\psi(t,i)-\sum_{j=1}^{d}q_{ij}\psi(t,j),\\ \psi(T,i)=1,\quad i=1,2,\ldots,d.\end{cases}\tag{3.3}$$

参考Ping和Yam(2013) [11]可知，$\psi(t,i)>0$且(3.3)的解存在且唯一.

定理1 若对每个$z>\psi(0,i_0)x_0$,

$$E\int_0^T[(r_1(t,i)-r_0(t,i)+\varepsilon_1(t,i))^2+(r_2(t,i)-r_0(t,i)+\varepsilon_2(t,i))^2]dt>0\tag{3.4}$$

成立, 则问题（3.2）可行.

证明 我们只需找到一个可容许策略满足$\mathbb{E}[X(T)]=z$, 假定投资组合为$\pi^\beta(t)=(\xi^\beta(t),l_1^\beta(t),l_2^\beta(t))$, 且$\xi^\beta(t)=\frac{\theta(t,i)}{\eta(t,i)}$, $l_1^\beta(t)=\beta(r_1(t,i)-r_0(t,i)+\varepsilon_1(t,i))\psi(t,i)$, $l_2^\beta(t)=\beta(r_2(t,i)-r_0(t,i)+\varepsilon_2(t,i))\psi(t,i)$, $\beta\in R$. 此时对应的财富过程用$X^\beta(t)$表示. 对$\psi(t,\alpha(t))$ 与$X^\beta(t)$的乘积运用$It\hat{o}$ 公式可得

$$\begin{aligned}
&d[\psi(t,\alpha(t))X^\beta(t)]\\
=&\{\dot{\psi}(t,\alpha(t))X^\beta(t)+\psi(t,\alpha(t))r_0(t,\alpha(t))X^\beta(t)\\
&+\sum_{j=1}^{d}q_{\alpha(t)j}\psi(t,j)X^\beta(t)\}dt+\beta[(r_1(t,\alpha(t))-r_0(t,\alpha(t))+\varepsilon_1(t,\alpha(t)))^2\\
&+(r_2(t,\alpha(t))-r_0(t,\alpha(t))+\varepsilon_2(t,\alpha(t)))^2]\psi^2(t,\alpha(t))\\
&+\{\cdots\}dW(t)+\{\cdots\}dW_0(t)+\{\cdots\}dW_1(t)+\{\cdots\}dW_2(t)\\
=&\beta[(r_1(t,\alpha(t))-r_0(t,\alpha(t))+\varepsilon_1(t,\alpha(t)))^2+(r_2(t,\alpha(t))-r_0(t,\alpha(t))\\
&+\varepsilon_2(t,\alpha(t)))^2]\psi^2(t,\alpha(t))+\{\cdots\}dW(t)+\{\cdots\}dW_0(t)+\{\cdots\}dW_1(t)+\{\cdots\}dW_2(t),
\end{aligned}\tag{3.5}$$

其中$\psi(t,i)$满足(3.3), 从0到T取积分求期望可得

$$\begin{aligned}
\mathbb{E}X^\beta(T)=&\mathbb{E}[\psi(T,\alpha(T))X^\beta(T)]\\
=&\psi(0,i_0)x_0+\beta\mathbb{E}\int_0^T\psi^2(t,\alpha(t))[(r_1(t,\alpha(t))-r_0(t,\alpha(t))+\varepsilon_1(t,\alpha(t)))^2\\
&+(r_2(t,\alpha(t))-r_0(t,\alpha(t))+\varepsilon_2(t,\alpha(t)))^2]dt.
\end{aligned}\tag{3.6}$$

由$\psi(t,i)>0$, 我们只需取

$$\beta=\frac{z-\psi(0,i_0)x_0}{\mathbb{E}\int_0^T\psi^2(t,\alpha(t))[(\mathcal{T}_1(t,\alpha(t)))^2+(\mathcal{T}_2(t,\alpha(t)))^2]dt}>0, \tag{3.7}$$

其中

$$\mathcal{T}_1(t,i)=r_1(t,i)-r_0(t,i)+\varepsilon_1(t,i),\quad \mathcal{T}_2(t,i)=r_2(t,i)-r_0(t,i)+\varepsilon_2(t,i), \tag{3.8}$$

则有$\mathbb{E}X^{\beta}(T)=z$, 且$\xi^{\beta}(t)<1, l_1^{\beta}(t)\geq 0, l_2^{\beta}(t)\geq 0$. 即$X^{\beta}(t)$是一个可容许策略.

容许可行解存在之后讨论均值方差最优问题. 问题(3.2)是有约束条件$\mathbb{E}[X(T)]=z$ 的凸最优化问题, 引入一个拉格朗日乘数$\lambda\in R$ 来消除约束条件$E[X(T)]=z$.

$$\begin{aligned} J(x_0,i_0,\pi,\lambda)&=\mathbb{E}\left\{[X(T)-z]^2\right\}+2\lambda[\mathbb{E}X(T)-z]\\ &=\mathbb{E}\left\{[X(T)+\lambda-z]^2\right\}-\lambda^2,\lambda\in R, \end{aligned} \tag{3.9}$$

那么, 式(3.2)求最优投资再保险策略问题可转化为求解参数为λ的随机控制问题

$$\begin{aligned} &\min\quad J(x_0,i_0,\pi,\lambda)=\mathbb{E}\left\{[X(T)+\lambda-z]^2\right\}-\lambda^2,\\ &\text{使得}\begin{cases}\pi\in\Pi,\\ (X(\cdot),\pi(\cdot))\ \text{满足}(2.10).\end{cases} \end{aligned} \tag{3.10}$$

这类问题我们称之为马尔科夫调制的随机线性二次型(LQ)最优控制问题.

问题(3.2)与(3.10)有相同的解. 首先考虑以下几个引理.

引理1 假设有二元函数$f(x,y)=ax^2+by^2+cx+dy+exy$, 若$a>0$且$4ab-e^2>0$, 则$f(x,y)$有极小值$f(x^*,y^*)=-\frac{bc^2+ad^2-cde}{4ab-e^2}$, 此时$(x^*,y^*)=\left(\frac{ed-2bc}{4ab-e^2},\frac{ec-2ad}{4ab-e^2}\right)$.

引理2 假设$K(t,i)$, $Q(t,i)$满足以下两组常微分方程

$$\begin{cases}\dot{K}(t,i)=[\mathcal{W}(t,i)+\frac{a^2(t,i)\eta^2(t,i)}{b^2(t,i)}-2r_0(t,i)]K(t,i)-\sum_{j=1}^d q_{ij}K(t,j),\\ K(T,i)=1,\quad i=1,2,\ldots,d,\end{cases} \tag{3.11}$$

$$\begin{cases}\dot{Q}(t,i)=r_0(t,i)Q(t,i)-\frac{a(t,i)(\theta(t,i)-\eta(t,i))}{\lambda-z}-\frac{1}{K(t,i)}\sum_{j=1}^d q_{ij}K(t,j)[Q(t,j)-Q(t,i)],\\ Q(T,i)=1,\quad i=1,2,\ldots,d.\end{cases} \tag{3.12}$$

其中

$$\mathcal{W}(t,i)=\frac{T_2(t,i)\mathcal{T}_1^2(t,i)+T_1(t,i)\mathcal{T}_2^2(t,i)-2\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)\mathcal{T}_1(t,i)\mathcal{T}_2(t,i)}{T_1(t,i)T_2(t,i)+\sigma_2^2(t,i))-\rho^2(t,i)\sigma_1^2(t,i)\sigma_2^2(t,i)},$$

$$T_1(t,i)=b_1^2(t,i)+\sigma_1^2(t,i),\quad T_2(t,i)=b_2^2(t,i)+\sigma_2^2(t,i).$$

由于式(3.11)(3.12)是线性微分方程且系数连续, 故解的存在唯一性显然. 此外, 与(3.3)证明类似可得$K(t,i)>0$. 式(3.11)是处理随机线性二次型控制问题的一般Riccati方程, 而(3.12)则为了解决(3.10)中的非齐次项.

定理2 问题(3.10)有最优反馈控制

$$\pi^*(t,x,i)=(\xi^*(t,x,i),l_1^*(t,x,i),l_2^*(t,x,i)),\tag{3.13}$$

其中

$$\xi^*(t,x,i)=1+\frac{a(t,i)\eta(t,i)[x+(\lambda-z)Q(t,i)]}{b^2(t,i)},$$

$$l_1^*(t,x,i)=\frac{\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)\mathcal{T}_2(t,i)-(b_2^2(t,i)+\sigma_2^2(t,i))\mathcal{T}_1(t,i)}{(b_1^2(t,i)+\sigma_1^2(t,i))(b_2^2(t,i)+\sigma_2^2(t,i))-\rho^2(t,i)\sigma_1^2(t,i)\sigma_2^2(t,i)}[x+(\lambda-z)Q(t,i)],$$

$$l_2^*(t,x,i)=\frac{\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)\mathcal{T}_1(t,i)-(b_1^2(t,i)+\sigma_1^2(t,i))\mathcal{T}_2(t,i)}{(b_1^2(t,i)+\sigma_1^2(t,i))(b_2^2(t,i)+\sigma_2^2(t,i))-\rho^2(t,i)\sigma_1^2(t,i)\sigma_2^2(t,i)}[x+(\lambda-z)Q(t,i)],$$

最优值

$$\begin{aligned}\inf_{\pi\in\Pi}J\left(x_0,i_0,\pi,\lambda\right)=&\left[K\left(0,i_0\right)Q^2\left(0,i_0\right)+\delta\left(i_0\right)-1\right](\lambda-z)^2\\&+2\left[K\left(0,i_0\right)Q\left(0,i_0\right)x_0-z\right](\lambda-z)+K\left(0,i_0\right)x_0^2-z^2,\end{aligned}\tag{3.14}$$

其中

$$\delta\left(i_0\right)=\sum_{i=1}^{d}\sum_{j=1}^{d}\int_0^T K_{i_0 i}q_{\alpha(t)j}[Q(t,j)-Q(t,i)]^2dt.$$

证明 对$d\left\{K(t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]^2\right\}$应用$It\hat{o}$ 引理

$$\begin{aligned}&d\{K\ (t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]^2\}\\&=\Big\{\dot{K}(t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]^2\\&\quad+2(\lambda-z)K(t,\alpha(t))\dot{Q}(t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]\\&\quad+2K(t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]\\&\quad\times[X(t)r_0(t,\alpha(t))+a(t,\alpha(t))(\theta(t,\alpha(t))-\eta(t,\alpha(t))\xi(t))\\&\quad+l_1(t)(r_1(t,\alpha(t))-r_o(t,\alpha(t))+\varepsilon_1(t,\alpha(t)))\\&\quad+l_2(t)(r_2(t,\alpha(t))-r_o(t,\alpha(t))+\varepsilon_2(t,\alpha(t)))]\\&\quad+K(t,\alpha(t))\left[b^2(t,\alpha(t))(1-\xi(t))^2\right.\\&\quad+b_1^2(t,\alpha(t))l_1^2(t)+b_2^2(t,\alpha(t))l_2^2(t)+\sigma_1^2(t,\alpha(t))l_1^2(t)+\sigma_2^2(t,\alpha(t))l_2^2(t)\\&\quad\left.+2\rho(t,\alpha(t))\sigma_1(t,\alpha(t))\sigma_2(t,\alpha(t))l_1(t)l_2(t)\right]\\&\quad+\sum_{j=1}^{d}q_{\alpha(t)j}K(t,j)[X(t)+(\lambda-z)Q(t,j)]^2\Big\}dt\\&\quad+\{\ldots\}dW(t)+\{\ldots\}dW_0(t)+\{\ldots\}dW_1(t)+\{\ldots\}dW_2(t)\end{aligned}$$

$$
\begin{aligned}
=&\{K(t,\alpha(t))b^2(t,\alpha(t))\left[\xi(t)-\frac{a(t,\alpha(t))\eta(t,\alpha(t))[x+(\lambda-z)Q(t,\alpha(t))]}{b^2(t,i)}-1\right]^2\\
&+l_1(t)[r_1(t,\alpha(t))-r_0(t,\alpha(t))+\varepsilon_1(t,\alpha(t))]2K(t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]\\
&+l_2(t)[r_1(t,\alpha(t))-r_0(t,\alpha(t))+\varepsilon_2(t,\alpha(t))]2K(t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]\\
&+K(t,\alpha(t))l_1^2(t)(\sigma_1^2(t,\alpha(t))+b_1^2(t,\alpha(t)))+K(t,\alpha(t))l_2^2(t)(\sigma_2^2(t,\alpha(t))+b_2^2(t,\alpha(t)))\\
&+2\rho(t,\alpha(t))K(t,\alpha(t))\sigma_1(t,\alpha(t))\sigma_2(t,\alpha(t))l_1(t)l_2(t)\\
&+\mathcal{W}(t,i)K(t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]^2\\
&-\sum_{j=1}^{d}q_{\alpha(t)j}K(t,j)(\lambda-z)^2Q(t,\alpha(t))^2+\sum_{j=1}^{d}q_{\alpha(t)j}K(t,j)(\lambda-z)^2Q(t,j)^2\\
&-2(\lambda-z)^2Q(t,\alpha(t))\sum_{j=1}^{d}q_{\alpha(t)j}K(t,j)[Q(t,j)-Q(t,\alpha(t))]+0\times X(t)^2+0\times X(t)\}dt\\
&+\{\ldots\}dW(t)+\{\ldots\}dW_0(t)+\{\ldots\}dW_1(t)+\{\ldots\}dW_2(t).
\end{aligned}\tag{3.15}
$$

从0到T取积分并求期望可得

$$
\begin{aligned}
&\mathbb{E}\int_0^T d\{K\ (t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]^2\}\\
&=\mathbb{E}\left\{[X(T)+\lambda-z]^2\right\}+K(0,i_0)[x_0+(\lambda-z)Q(0,i_0)]^2,
\end{aligned}\tag{3.16}
$$

我们可以先求得

$$
\begin{aligned}
&\mathbb{E}\left\{[X(T)+\lambda-z]^2\right\}-\lambda^2\\
&=\mathbb{E}\int_0^T d\{K\ (t,\alpha(t))[X(t)+(\lambda-z)Q(t,\alpha(t))]^2\}+K(0,i_0)[x_0+(\lambda-z)Q(0,i_0)]^2-\lambda^2\\
&\geq-\sum_{j=1}^{d}q_{\alpha(t)j}K(t,j)(\lambda-z)^2Q(t,\alpha(t))^2+\sum_{j=1}^{d}q_{\alpha(t)j}K(t,j)(\lambda-z)^2Q(t,j)^2\\
&\quad-2(\lambda-z)^2Q(t,\alpha(t))\sum_{j=1}^{d}q_{\alpha(t)j}K(t,j)[Q(t,j)-Q(t,\alpha(t))]+K(0,i_0)[x_0+(\lambda-z)Q(0,i_0)]^2-\lambda^2\\
&=(\lambda-z)^2\mathbb{E}\int_0^T\left\{\sum_{j=1}^{d}q_{\alpha(t)j}K(t,j)[Q(t,\alpha(t))-Q(t,j)]^2\right\}+K(0,i_0)[x_0+(\lambda-z)Q(0,i_0)]^2-\lambda^2,
\end{aligned}\tag{3.17}
$$

即最优值

$$
\begin{aligned}
\inf_{\pi\in\Pi}J\left(x_0,i_0,\pi,\lambda\right)=&\left[K\left(0,i_0\right)Q^2\left(0,i_0\right)+\delta\left(i_0\right)-1\right]\left(\lambda-z\right)^2\\
&+2\left[K\left(0,i_0\right)Q\left(0,i_0\right)x_0-z\right]\left(\lambda-z\right)+K\left(0,i_0\right)x_0^2-z^2.
\end{aligned}\tag{3.18}
$$

其中

$$\delta\left(i_0\right)=\sum_{i=1}^{d}\sum_{j=1}^{d}\int_0^T p_{i_0 i}(t)q_{\alpha(t)j}[Q(t,j)-Q(t,i)]^2dt. \tag{3.19}$$

由于要满足$\pi^*\in\Pi$, 接下来讨论所得(3.13)π^*可容许性. 首先令

$$M_1(t,i)=\frac{\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)\mathcal{T}_2(t,i)-(b_2^2(t,i)+\sigma_2^2(t,i))\mathcal{T}_1(t,i)}{(b_1^2(t,i)+\sigma_1^2(t,i))(b_2^2(t,i)+\sigma_2^2(t,i))-\rho^2(t,i)\sigma_1^2(t,i)\sigma_2^2(t,i)}, \tag{3.20}$$

$$M_2(t,i)=\frac{\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)\mathcal{T}_1(t,i)-(b_1^2(t,i)+\sigma_1^2(t,i))\mathcal{T}_2(t,i)}{(b_1^2(t,i)+\sigma_1^2(t,i))(b_2^2(t,i)+\sigma_2^2(t,i))-\rho^2(t,i)\sigma_1^2(t,i)\sigma_2^2(t,i)}, \tag{3.21}$$

若考虑$M_1(t,i)=0$且$M_2(t,i)=0$有

$$\frac{b_1^2(t,i)b_2^2(t,i)+b_1^2(t,i)\sigma_2^2(t,i)+\sigma_1^2(t,i)b_2^2(t,i)+\sigma_1^2(t,i)\sigma_2^2(t,i)}{\sigma_1^2(t,i)\sigma_2^2(t,i)\rho^2(t,i)}\neq 1,$$

故$M_1(t,i)$, $M_2(t,i)$不能同时为零.

我们讨论$M_1(t,i)$, $M_2(t,i)$可满足条件, 由于推导比较冗长, 经过整理直接给出结果,

情形1

$$\mathcal{T}_1(t,i)\leq\frac{\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)\mathcal{T}_2(t,i)}{b_2^2(t,i)+\sigma_2^2(t,i)}, \tag{3.22}$$

情形2

$$\mathcal{T}_1(t,i)\geq\frac{(b_1^2(t,i)+\sigma_1^2(t,i))\mathcal{T}_2(t,i)}{\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)}, \tag{3.23}$$

情形3

$$\frac{\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)\mathcal{T}_2(t,i)}{b_2^2(t,i)+\sigma_2^2(t,i)}<\mathcal{T}_1(t,i)<\frac{(b_1^2(t,i)+\sigma_1^2(t,i))\mathcal{T}_2(t,i)}{\sigma_1(t,i)\sigma_2(t,i)\rho(t,i)}. \tag{3.24}$$

其中情形3最优策略可见定理2.

引理3 考虑最优反馈控制(3.13), 若$K(0,i)[x_0+(\lambda-z)Q(0,i)]$, $i=1,2,\ldots,d$正负一致. 那么, $K(0,\alpha(t))[X(t)+(\lambda-z)Q(0,\alpha(t))], t\in[0,T]$的正负由$K(0,i_0)[x_0+(\lambda-z)Q(0,i_0)]$的正负决定.

引理4 考虑最优反馈控制(3.13), $X(t)+(\lambda-z)Q(0,\alpha(t))<0, t\in[0,T]$当且仅当$x_0+(\lambda-z)Q(0,i_0)<0$, $i_0=1,2,\ldots,d$.

引理3和引理4证明过程可参考Ping和Yam (2013) [11].

定理3 考虑情形1, 若$x_0+(\lambda-z)Q_1(0,i_0)<0$, $i_0=1,2,\ldots,d$, 其中$K_1(t,i)$, $Q_1(t,i)$满足以下常微分方程组

$$\begin{cases}\dot{K}_1(t,i)=[\frac{(r_2(t,i)-r_0(t,i)+\varepsilon_2(t,i))^2}{b_2^2(t,i)+\sigma_2^2(t,i)}+\frac{a^2(t,i)\eta^2(t,i)}{b^2(t,i)}-2r_0(t,i)]K_1(t,i)-\sum_{j=1}^d q_{ij}K_1(t,j),\\ K_1(T,i)=1,\quad i=1,2,\ldots,d.\end{cases} \tag{3.25}$$

$$\begin{cases} \dot{Q}_1(t,i) = r_0(t,i)Q_1(t,i) - \frac{a(t,i)(\theta(t,i)-\eta(t,i))}{\lambda-z} - \frac{1}{K_1(t,i)}\sum_{j=1}^{d} q_{ij}K_1(t,j)[Q_1(t,j)-Q_1(t,i)], \\ Q_1(T,i) = 1, \quad i=1,2,\ldots,d. \end{cases} \tag{3.26}$$

则问题(3.10)最优反馈控制

$$\tilde{\pi}^*(t,x,i) = (\tilde{\xi}^*(t,x,i), \tilde{l_1}^*(t,x,i), \tilde{l_2}^*(t,x,i)), \tag{3.27}$$

其中

$$\tilde{\xi}^*(t,x,i) = 1 + \frac{a(t,i)\eta(t,i)[x+(\lambda-z)Q_1(t,i)]}{b^2(t,i)}, \quad \tilde{l_1}^*(t,x,i) = 0,$$

$$\tilde{l_2}^*(t,x,i) = -\frac{(r_2(t,i)-r_0(t,i)+\varepsilon_2(t,i))}{b_2^2(t,i)+\sigma_2^2(t,i)}[x+(\lambda-z)Q_1(t,i)],$$

最优值

$$\begin{aligned} \inf_{\pi\in\Pi} J(x_0,i_0,\pi,\lambda) = & \left[K_1(0,i_0)Q_1^2(0,i_0) + \delta_1(i_0) - 1\right](\lambda-z)^2 \\ & + 2\left[K_1(0,i_0)Q_1(0,i_0)x_0 - z\right](\lambda-z) + K_1(0,i_0)x_0^2 - z^2, \end{aligned} \tag{3.28}$$

其中

$$\delta_1(i_0) = \sum_{i=1}^{d}\sum_{j=1}^{d}\int_0^T p_{i_0 i}(t) q_{\alpha(t)j}[Q_1(t,j)-Q_1(t,i)]^2 dt.$$

证明过程类似于定理2.

定理4 考虑情形2, 若$x_0 + (\lambda - z)Q_2(0,i_0) < 0$，$i_0 = 1,2,\ldots,d$, 其中$K_2(t,i)$，$Q_2(t,i)$满足以下常微分方程组

$$\begin{cases} \dot{K}_2(t,i) = [\frac{(r_1(t,i)-r_0(t,i)+\varepsilon_1(t,i))^2}{b_1^2(t,i)+\sigma_1^2(t,i)} + \frac{a^2(t,i)\eta^2(t,i)}{b^2(t,i)} - 2r_0(t,i)]K_2(t,i) - \sum_{j=1}^{d} q_{ij}K_2(t,j), \\ K_2(T,i) = 1, \quad i=1,2,\ldots,d, \end{cases} \tag{3.29}$$

$$\begin{cases} \dot{Q}_2(t,i) = r_0(t,i)Q_2(t,i) - \frac{a(t,i)(\theta(t,i)-\eta(t,i))}{\lambda-z} - \frac{1}{K_2(t,i)}\sum_{j=1}^{d} q_{ij}K_2(t,j)[Q_2(t,j)-Q_2(t,i)], \\ Q_2(T,i) = 1, \quad i=1,2,\ldots,d, \end{cases} \tag{3.30}$$

则问题(3.10)有最优反馈控制

$$\bar{\pi}^*(t,x,i) = (\bar{\xi}^*(t,x,i), \bar{l_1}^*(t,x,i), \bar{l_2}^*(t,x,i)), \tag{3.31}$$

其中

$$\bar{\xi}^*(t,x,i) = 1 + \frac{a(t,i)\eta(t,i)[x+(\lambda-z)Q_2(t,i)]}{b^2(t,i)}, \quad \bar{l_2}^*(t,x,i) = 0,$$

$$\bar{l_1}^*(t,x,i) = -\frac{r_1(t,i)-r_0(t,i)+\varepsilon_1(t,i)}{b_1^2(t,i)+\sigma_1^2(t,i)}[x+(\lambda-z)Q_2(t,i)],$$

最优值

$$\inf_{\pi\in\Pi} J(x_0,i_0,\pi,\lambda)=\left[K_2(0,i_0)Q_2^2(0,i_0)+\delta_2(i_0)-1\right](\lambda-z)^2 \\ +2\left[K_2(0,i_0)K_2(0,i_0)x_0-z\right](\lambda-z)+K_2(0,i_0)x_0^2-z^2, \tag{3.32}$$

其中

$$\delta_2(i_0)=\sum_{i=1}^{d}\sum_{j=1}^{d}\int_0^T p_{i_0 i}(t)q_{\alpha(t)j}[Q_2(t,j)-Q_2(t,i)]^2dt.$$

证明过程类似于定理2.

4. 有效前沿

我们已经研究了问题(3.10)的解, 本节讨论(3.2)的最优策略和有效前沿问题, 仅讨论情形3且$x_0+(\lambda-z)Q(0,i_0)<0$, 其它情形可类似得到结果. 若存在至少一个策略$\pi$满足(3.2)所有约束条件, 我们称$\pi$是有效的, 称$(VarX(T),z)\in R^2$是有效点, 而所有有效点的集合则称为有效前沿. 应用著名的拉格朗日对偶定理

$$J^*(x_0,i_0)=\sup_{\lambda\in\mathbb{R}}\inf_{\pi\in\Pi}J(x_0,i_0,\pi,\lambda)<\infty, \tag{4.1}$$

由于$\inf_{\pi\in\Pi}J(x_0,i_0,\pi,\lambda)$是关于$\lambda-z$ 的二次函数, 而$Q(0,i_0)$是λ的函数, 故将$Q(t,i)$ 做以下分解

$$(\lambda-z)Q(t,i)=(\lambda-z)\tilde{Q}(t,i)+\bar{Q}(t,i), \tag{4.2}$$

其中$\tilde{Q}(t,i)$, $\bar{Q}(t,i)$满足

$$\begin{cases}\dot{\tilde{Q}}(t,i)=r_0(t,i)\tilde{Q}(t,i)-\frac{1}{K(t,i)}\sum_{j=1}^d q_{ij}K(t,j)[\tilde{Q}(t,j)-\tilde{Q}(t,i)],\\ \tilde{Q}(T,i)=1,\quad i=1,2,\ldots,d,\end{cases} \tag{4.3}$$

$$\begin{cases}\dot{\bar{Q}}(t,i)=r_0(t,i)\bar{Q}(t,i)-a(t,i)\left[\theta(t,i)-\eta(t,i)\right]-\frac{1}{K(t,i)}\sum_{j=1}^d q_{ij}K(t,j)[\bar{Q}(t,j)-\bar{Q}(t,i)],\\ \bar{Q}(T,i)=1,\quad i=1,2,\ldots,d.\end{cases} \tag{4.4}$$

此时$\tilde{Q}(t,i)$, $\bar{Q}(t,i)$不再是λ的函数, 将(4.2)代入(3.14)有

$$\begin{aligned}&\inf_{\pi\in\Pi}J(x_0,i_0,\pi,\lambda)\\ &=\left\{K(0,i_0)\left[\tilde{Q}(0,i_0)+\frac{\bar{Q}(0,i_0)}{\lambda-z}\right]^2+\gamma(i_0)+\frac{2}{\lambda-z}\mu(i_0)+\frac{1}{(\lambda-z)^2}\nu(i_0)-1\right\}(\lambda-z)^2\\ &\quad+2\left\{K(0,i_0)\left[\tilde{Q}(0,i_0)+\frac{\bar{Q}(0,i_0)}{\lambda-z}\right]x_0-z\right\}(\lambda-z)+K(0,i_0)x_0^2-z^2\\ &=\left[K(0,i_0)\tilde{Q}^2(0,i_0)+\gamma(i_0)-1\right](\lambda-z)^2+K(0,i_0)\left[x_0+\bar{Q}(0,i_0)\right]^2-z^2+\nu(i_0)\\ &\quad+\left\{2K(0,i_0)\left[x_0+\bar{Q}(0,i_0)\right]\tilde{Q}(0,i_0)+2\mu(i_0)-2z\right\}(\lambda-z),\end{aligned} \tag{4.5}$$

其中

$$\gamma(i_0)=\mathbb{E}\int_0^T\sum_{j=1}^d q_{\alpha(t)j}[\tilde{Q}(t,j)-\tilde{Q}(t,i)]^2dt,\quad \nu(i_0)=\mathbb{E}\int_0^T\sum_{j=1}^d q_{\alpha(t)j}[\bar{Q}(t,j)-\bar{Q}(t,i)]^2dt.$$

$$\mu(i_0)=\mathbb{E}\int_0^T\sum_{j=1}^d q_{\alpha(t)j}[\tilde{Q}(t,j)-\tilde{Q}(t,i)][\bar{Q}(t,j)-\bar{Q}(t,i)]dt,$$

若可行性成立, 则有

$$K(0,i_0)\,\tilde{Q}^2(0,i_0)+\gamma(i_0)-1<0, \tag{4.6}$$

取最优策略时λ^*-z有

$$\lambda^*-z=\frac{z-K(0,i_0)\,\tilde{Q}(0,i_0)\bar{Q}(0,i_0)-K(0,i_0)\,\tilde{Q}(0,i_0)x_0-\mu(i_0)}{K(0,i_0)\,\tilde{Q}^2(0,i_0)+\gamma(i_0)-1}, \tag{4.7}$$

最大值为

$$\begin{aligned}
&VarX^*(T)\\
=&\frac{\gamma(i_0)+K(0,i_0)\,\tilde{Q}^2(0,i_0)}{1-\gamma(i_0)-K(0,i_0)\,\tilde{Q}^2(0,i_0)}\left\{z-\frac{K(0,i_0)\,\tilde{Q}(0,i_0)\left[\bar{Q}(0,i_0)+x_0\right]+\mu(i_0)}{\gamma(i_0)+K(0,i_0)\,\tilde{Q}^2(0,i_0)}\right\}^2\\
&+\frac{K(0,i_0)\left[\bar{Q}(0,i_0)+x_0\right]^2\gamma(i_0)-2\mu(i_0)\,K(0,i_0)\,\tilde{Q}(0,i_0)\left[\bar{Q}(0,i_0)+x_0\right]-\mu^2(i_0)}{\gamma(i_0)+K(0,i_0)\,\tilde{Q}^2(0,i_0)}\\
&+\nu(i_0)\,.
\end{aligned} \tag{4.8}$$

5. 小结

本文主要研究了在马尔科夫调制的市场状态下, 以均值方差为准则的最优投资再保险策略问题. 假定在马尔科夫调制下, 保险公司购买再保险, 盈余投资到一个无风险资产和两个风险资产, 风险资产相依性由跳过程相依来刻画, 采用随机线性二次型技术和拉格朗日对偶定理, 求出了基于两个常微分方程组的最优投资再保险策略以及有效前沿的解, 我们发现由于马尔科夫调制的作用, 风险永远不能被消除. 此外, 本文只考虑了比例再保险, 而保费的计算有多种方法, 且在现实生活中, 金融市场会存在交易费用, 也会涉及到分红、资产负债等情形, 这些问题仍值得我们做更深入的研究.

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参考文献

[1] Markowitz, H.M. (1952) Portfolio Selection. *The Journal of Finance*, **7**, 77-91. https://doi.org/10.2307/2975974

[2] Bäuerle, N. (2005) Benchmark and Mean-Variance Problems for Insurers. *Mathematical Methods of Operations Research*, **62**, 159-165. https://doi.org/10.1007/s00186-005-0446-1

[3] Bai, L. and Zhang, H. (2008) Dynamic Mean-Variance Problem with Constrained Risk Control for the Insurers. *Mathematical Methods of Operational Research*, **68**, 181-205. https://doi.org/10.1007/s00186-007-0195-4

[4] Yan, Z. and Li, Z. (2011) Optimal Time-Consistent Investment and Reinsurance Policies for Mean-Variance Insurers. *Insurance Mathematics and Economics*, **49**, 145-154. https://doi.org/10.1016/j.insmatheco.2011.01.001

[5] Bi, J., Liang, Z. and Yuen, K.C. (2019) Optimal Mean-Variance Investment/Reinsurance with Common Shock in a Regime-Switching Market. *Mathematical Methods of Operations Research*, 90, 109-135. https://doi.org/10.1007/s00186-018-00657-3

[6] Promislow, S.D. and Young, V.R. (2005) Minimizing the Probability of Ruin When Claims Follow Brownian Motion with Drift. *North American Actuarial Journal*, **9**, 110-128. https://doi.org/10.1080/10920277.2005.10596214

[7] Bi, J. and Guo, J. (2013) Optimal Mean-Variance Problem with Constrained Controls in a Jump-Diffusion Financial Market for an Insurer. *Journal of Optimization Theory and Applications*, **157**, 252-275. https://doi.org/10.1007/s10957-012-0138-y

[8] Liang, Z., Yuen, K.C. and Guo, U. (2011) Optimal Proportional Reinsurance and Investment in a Stock Market with Ornstein-Uhlenbeck Process. *Insurance: Mathematics and Economics*, **49**, 207-215. https://doi.org/10.1016/j.insmatheco.2011.04.005

[9] Hamilton, J.D. (1989) A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, **57**, 357-384. https://doi.org/10.2307/1912559

[10] Zhou, X.Y. and Yin, G. (2003) Markowitz's Mean-Variance Portfolio Selection with Regime Switching: A Continuous-Time Model. *SIAM Journal on Control and Optimization*, **42**, 1466-1482. https://doi.org/10.1137/S0363012902405583

[11] Ping, C. and Yam, S. (2013) Optimal Proportional Reinsurance and Investment with Regime-Switching for Mean-Variance Insurers. *Insurance Mathematics and Economics*, **53**, 871-883. https://doi.org/10.1016/j.insmatheco.2013.10.004

[12] 张彩斌, 梁志彬, 袁锦泉. Markov调节中基于时滞和相依风险模型的最优再保险与投资[J]. 中国科学: 数学, 2021, 51(5): 773-796.

[13] Grandell, J. (1991) Aspects of Risk Theory. World Publishing Co., New York. https://doi.org/10.1007/978-1-4613-9058-9