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一类含参量拟线性微分系统正解的存在唯一性

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摘 要

本文利用不动点定理研究了一类含有两个参数的拟线性微分系统

$$\begin{cases} -((u')^{p-1})' = \lambda f(t,u(t),v(t)), \quad t\in(0,1), \\ -((v')^{q-1})' = \mu g(t,u(t),v(t)), \\ u(0)=u'(1)=0, \\ v(0)=v'(1)=0 \end{cases}$$

正解的存在唯一性，其中 $p,q>1$, $f,g:[0,1]\times[0,+\infty)\times[0,+\infty)\to[0,+\infty)$连续。对于任意固定的 λ, $\mu>0$, f,g满足规定的条件时，得到系统正解的存在唯一性。最后，举例说明结论的可行性。

关键词

微分系统，锥，正解，拟线性

Existence and Uniqueness of Positive Solutions for a Class of Quasilinear Differential Systems with Parameters

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Abstract

In this paper, by using a new fixed point theorem to study existence and uniqueness of positive solutions for a class of quasilinear differential systems with parameters

$$\begin{cases} -((u')^{p-1})' = \lambda f(t, u(t), v(t)), \quad t \in (0,1), \\ -((v')^{q-1})' = \mu g(t, u(t), v(t)), \\ u(0) = u'(1) = 0, \\ v(0) = v'(1) = 0, \end{cases}$$

where $f, g : [0,1] \times [0,+\infty) \times [0,+\infty) \to [0,+\infty)$ are continuous, λ, and μ are positive parameters, we establish sufficient conditions for the existence and uniqueness of positive solutions to this system for any fixed $\lambda, \mu > 0$. Finally, we give a simple example to illustrate our main result.

Keywords

Differential System, Cone, Positive Solution, Quasilinear

1. 介绍

微分系统越来越多地用于描述经济, 种群动态, 控制, 生态学和流行病等领域的许多问题. 由于其深厚的现实背景和重要作用, 越来越受到人们的重视. 尤其是拟线性微分系统边值问题, 它在物理学, 生物学等领域有着广泛的应用. 通过查阅文献我们知道, 对于拟线性微分系统的研究相对较少 [1–8]. 在文献 [9]中, 杨志林等人利用不动点指数研究同时包含 p-Laplacian 和一阶导数的方程

$$\begin{cases} -((u')^{p-1})' = f(t, u, u'), \quad t \in (0,1), \\ u(0) = u'(1) = 0 \end{cases} \tag{1.1}$$

多解的存在性, 基于詹森不等式和其他的不等式得到了正解的存在性. 其中 $p > 1$, $f \in C^1([0,1] \times R_+ \times R_+, R_+)(R_+ := [0,\infty)), u \in C^2([0,1], R) \cap C^1([0,1], R), t \in (0,1)$.

在文献 [10]中, 杨志林利用不动点指数研究了微分系统

$$\begin{cases} -((u_i')^{p-1})' = f_i(t, u_1, ..., u_n), & t \in (0,1), \\ u_i(0) = u_i'(1) = 0 \end{cases} \tag{1.2}$$

多解的存在性, 与问题 (1.1) 相比, (1.2) 则是利用詹森不等式和非负矩阵得到正解的存在性, $n \geq 2, p_i > 1, f_i \in C([0,1] \times R_+^n, R_+)(i = 1, 2, ..., n, R_+ := [0, \infty))$.

最近在文献 [8]中, 杨志林等人利用不动点指数研究了二阶拟线性微分系统

$$\begin{cases} -((u')^{p-1})' = f(t, u(t), v(t)), & t \in (0,1), \\ -((v')^{q-1})' = g(t, u(t), v(t)), & t \in (0,1), \\ u(0) = u'(1) = 0, \\ v(0) = v'(1) = 0 \end{cases} \tag{1.3}$$

正解的存在性, 与问题 (1.1), (1.2) 不同的是, 在问题 (1.3) 中是利用关于非负凹函数和齐次算子得到正解的存在性, 其中 $p, q > 1,\ f,\ g : [0,1] \times [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ 连续.

问题 (1.1)-(1.3) 均未涉及参数, 且都用不动点指数理论研究微分系统正解的存在性, 受上述文献的启发, 本文利用不动点定理研究含两个参数的拟线性微分系统

$$\begin{cases} -((u')^{p-1})' = \lambda f(t, u(t), v(t)), & t \in (0,1), \\ -((v')^{q-1})' = \mu g(t, u(t), v(t)), \\ u(0) = u'(1) = 0, \\ v(0) = v'(1) = 0 \end{cases} \tag{1.4}$$

正解的存在唯一性, 其中 $p, q > 1,\ \lambda, \mu > 0,\ f, g : [0,1] \times [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ 连续.

2. 预备知识

本文的主要定义和引理:

定义 2.1. [11] 设 $(E, ||.||)$ 是实 Banach 空间, $P \in E$ 是一个锥, 如果 P 满足

(i) $\forall p \in E$, 和 $\lambda \geq 0$, 都有 $\lambda p \in P$;

(ii) 若 $-x \in P$, 则 $x = \Theta_E$, 其中 Θ_E 是 Banach 空间 E 中的零元素.

定义 2.2. [11] 设 $(E, ||.||)$ 是实 Banach 空间, $P \in E$ 是一个锥, 当 $\forall x, y \in E, y \geq x$时, 则 $y - x \in P$.

定义 2.3. [12] 如果满足

$$\forall x, y \in P,\ \Theta_E \leq x \leq y, \exists N > 0, ||x|| \leq N||y||,$$

则称 $P \in E$ 是一个正规锥.

定义 2.4. [12] 对 $\forall x, y \in E,\ x \le y$, 有 $Ax \le Ay$, 则称 A 是增算子.

定义等价关系 $x \sim y$, 即存在常数 $\alpha, \beta > 0$, 使得 $\alpha y \le x \le \beta y$. $P_h = \{x \in E, x \sim h\}$, 其中 $h > \Theta_E$. 易

知 $P_h \subset P$. 对任意 $h_1, h_2 \in P, h_1, h_2 \ne \Theta_E$, 令 $h = (h_1, h_2) \in \bar{P}_h = P \times P$, 如果 P 是正规锥, 则 $\bar{P}_h = (P, P)$ 是正规锥.

设 $\Phi = \{\varphi(r) \in (0,1) : \varphi(r) > r, r \in (0,1)\}$.

引理 2.5. [13] $\bar{P}_h = \{(u,v) : u \in P_{h_1}, v \in P_{h_2}\} = P_{h_1} \times P_{h_2}$.

引理 2.6. [14] 设 P 是 Banach 空间 E 中的一个正规锥, 对任意的 $h = (h_1, h_2) \in P \times P$, 其中 $h_1, h_2 \ne \Theta$. 算子

$A, B : P \times P \to P$ 是增算子, 且满足下列条件

(C_1) 存在 $\varphi_1, \varphi_2 \in \Phi$ 使得

$$A(ru, rv) \ge \varphi_1(r)A(u,v),\ B(ru, rv) \ge \varphi_2(r)B(u,v),\ r \in (0,1), u, v \in P;$$

(C_2) 存在 $(c_1, c_2) \in \bar{P}_h$, 使得 $A(c_1, c_2) \in P_{h_1},\ B(c_1, c_2) \in P_{h_2}$.

则

(a) $A : P_{h_1} \times P_{h_2} \to P_{h_1},\ B : P_{h_1} \times P_{h_2} \to P_{h_2}$, 且存在 $u_1, v_1 \in P_{h_1},\ u_2, v_2 \in P_{h_2}, r \in (0,1)$ 使得

$$r(v_1, v_2) \le (u_1, u_2) \le (v_1, v_2),\ u_1 \le A((u_1, u_2)) \le v_1,\ u_2 \le B((u_1, u_2)) \le v_2;$$

(b) 对任意固定的 λ, μ, 算子方程 $(u, v) = (\lambda A(u, v), \mu B(u, v))$ 有唯一的不动点 $(u\hat{*}_{\lambda,\mu}, v\hat{*}_{\lambda,\mu}) \in \bar{P}_h$, 另外对于

任意的初始点 $(u_0, v_0) \in \bar{P}_h$, 有序列

$$(u_n, v_n) = (\lambda A(u_{n-1}, v_{n-1}), \mu B(u_{n-1}, v_{n-1})),\ n = 1, 2, ...$$

且满足 $||u_n - u\hat{*}_{\lambda,\mu}|| \to 0, ||v_n - v\hat{*}_{\lambda,\mu}|| \to 0,\ \ n \to \infty$.

3. 主要结果及其证明

本文的工作空间是实 Banach 空间 $E = C[0,1]$, 范数为 $||u|| = \max\{|u(t)| : t \in [0,1]\}$, 记锥 $P = \{u \in E : u(t) \ge 0, t \in [0,1]\}$, 则 $P \subset E$. 且易知当正规常数 $N = 1$ 时, P 是正规锥.

定义

$$||(u,v)|| = ||u|| + ||v||,\ (u,v) \in E^2,$$

其中 $E^2 = E \times E$ 是定义在上述范数下的实 Banach 空间, 且 $P^2 \subset E^2$.

$$\bar{P}_h = \{(u,v) \in E \times E : u(t) \ge 0, v(t) \ge 0, t \in [0,1]\},$$

知 $\bar{P}_h \subset E \times E$, 由于 P 是正规锥, 则 $\bar{P}_h = P \times P$ 是正规锥. 在 $E \times E$ 上有下列序关系, 若$u_1(t) \le u_2(t),\ v_1(t) \le v_2(t),\ t \in [0,1]$, 则 $(u_1,\ v_1) \le (u_2,\ v_2)$.

微分系统 (1.4) 有解当且仅当积分方程组

$$\begin{cases} u(t) = \int_0^t (\int_s^1 \lambda f(\tau, u(\tau), v(\tau)) d\tau)^{p-1} ds, \\ v(t) = \int_0^t (\int_s^1 \mu g(\tau, u(\tau), v(\tau)) d\tau)^{q-1} ds \end{cases} \tag{3.1}$$

有解.

定义如下算子 $A_1,\ A_2 : P^2 \to P$, $A : P^2 \to P^2$

$$A_1(u,v)(t) = \lambda^{p-1} \int_0^t (\int_s^1 f(\tau, u(\tau), v(\tau)) d\tau)^{p-1} ds,$$

$$A_2(u,v)(t) = \mu^{q-1} \int_0^t (\int_s^1 g(\tau, u(\tau), v(\tau)) d\tau)^{q-1} ds,$$

$$A(u,v)(t) = (\lambda^{p-1} A_1(u,v)(t), \mu^{q-1} A_2(u,v)(t)). \tag{3.2}$$

则 $A_1, A_2 : P^2 \to P$ 和 $A : P^2 \to P^2$. 显然微分系统 (1.4) 的可解性等价于算子方程 A 有不动点.

记

$$h_1(t) = \int_0^t (1-s)^{p-1} ds,\ \ h_2(t) = \int_0^t (1-s)^{q-1} ds,\ t \in [0,1],$$

易知 $h_1(t), h_2(t) \ge 0, t \in [0,1]$, 则 $h_1, h_2 \in P$.

记

$$l_1 = \min_{t\in[0,1]} \int_0^t (1-s)^{p-1} ds,\ l_2 = \min_{t\in[0,1]} \int_0^t (1-s)^{q-1} ds,$$

$$L_1 = \max_{t\in[0,1]} \int_0^t (1-s)^{p-1} ds,\ L_2 = \max_{t\in[0,1]} \int_0^t (1-s)^{q-1} ds.$$

显然

$$l_1 \le h_1(t) \le L_1,\ l_2 \le h_2(t) \le L_2.$$

本文的主要结果是

定理 3.1.设 $h_1, h_2 \in P$, 假设

(H_1) $f, g \in C([0,1] \times R_+ \times R_+, R_+)$, 且 $f(t, l_1, l_2) > 0,\ g(t, l_1, l_2) > 0, t \in [0,1]$;

(H_2) f, g 关于第二和第三变量递增, 即对任意 $0 \le u_1 \le u_2,\ 0 \le v_1 \le v_2,\ t \in [0,1]$,

$$f(t, u_1, v_1) \le f(t, u_2, v_2),\ g(t, u_1, v_1) \le g(t, u_2, v_2);$$

(H_3) 存在 $\varphi_1, \varphi_2 \in \Phi$, $\forall u, v \in R_+, t \in [0,1], r \in (0,1)$ 使得

$$f(t, ru, rv) \ge \varphi_1^{p-1} f(t, u, v),\ g(t, ru, rv) \ge \varphi_2^{q-1} g(t, u, v).$$

则

(a) 存在 $u_1, v_1 \in P_{h_1}, u_2, v_2 \in P_{h_2}, r \in (0,1)$, 使得 $r(v_1, v_2) \leq (u_1, u_2) \leq (v_1, v_2)$, 且

$$u_1 \leq \int_0^t (\int_s^1 f(\tau, u(\tau), v(\tau))d\tau)^{p-1} ds \leq v_1, t \in [0,1],$$

$$u_2 \leq \int_0^t (\int_s^1 g(\tau, u(\tau), v(\tau))d\tau)^{q-1} ds \leq v_2, t \in [0,1].$$

(b) 对任意固定的 $\lambda, \mu > 0, t \in [0,1]$, 系统(1.4)有唯一的正解 $(u^*_{\lambda,\mu}, v^*_{\lambda,\mu}) \in \bar{P}_h$;

(c) 对任意初始点 $(u_0, v_0) \in \bar{P}_h$, 满足

$$u_{n+1} = \int_0^t (\int_s^1 \lambda f(\tau, u_n(\tau), v_n(\tau))d\tau)^{p-1} ds, n = 1, 2, ...$$

$$v_{n+1} = \int_0^t (\int_s^1 \mu g(\tau, u_n(\tau), v_n(\tau))d\tau)^{q-1} ds, n = 1, 2, ...$$

当 $n \to \infty, u_n \to u^*_{\lambda,\mu}, v_n \to v^*_{\lambda,\mu}$.

证明 首先证 A_1, A_2 是增算子. 对任意的 $u_i, v_i \in P, i = 1, 2, u_1 \leq u_2, v_1 \leq v_2$, 即 $u_1(t) \leq u_2(t), v_1(t) \leq v_2(t)$, 由 (H_2) 知

$$\begin{aligned} A_1(u_1, v_1)(t) &= \int_0^t (\int_s^1 f(\tau, u_1(\tau), v_1(\tau))d\tau)^{p-1} \\ &\leq \int_0^t (\int_s^1 f(\tau, u_2(\tau), v_2(\tau))d\tau)^{p-1} \\ &= A_1(u_2, v_2)(t), \end{aligned} \tag{3.3}$$

$$\begin{aligned} A_2(u_1, v_1)(t) &= \int_0^t (\int_s^1 g(\tau, u_1(\tau), v_1(\tau))d\tau)^{q-1} \\ &\leq \int_0^t (\int_s^1 g(\tau, u_2(\tau), v_2(\tau))d\tau)^{q-1} \\ &= A_2(u_2, v_2)(t). \end{aligned} \tag{3.4}$$

由 (3.3), (3.4) 知 $A_1(u_1, v_1) \leq A_1(u_2, v_2), A_2(u_1, v_1) \leq A_2(u_2, v_2)$.

对任意 $u, v \in P, r \in (0,1)$, 由 (H_3) 可得

$$\begin{aligned} A_1(ru, rv)(t) &= \int_0^t (\int_s^1 f(\tau, ru(\tau), rv(\tau))d\tau)^{p-1} \\ &\geq \varphi_1(r) \int_0^t (\int_s^1 f(\tau, u(\tau), v(\tau))d\tau)^{p-1} \\ &= \varphi_1 A_1(u, v)(t), \end{aligned}$$

$$
\begin{aligned}
A_2(ru,rv)(t) &= \int_0^t (\int_s^1 g(\tau,ru(\tau),rv(\tau))d\tau)^{q-1} \\
&\geq \varphi_2(r)\int_0^t (\int_s^1 g(\tau,u(\tau),v(\tau))d\tau)^{q-1} \\
&= \varphi_2(t)A_2(u,v)(t),
\end{aligned}
$$

即 $\forall u,v\in P, r\in(0,1)$, $A_1(ru,rv)(t)\geq\varphi_1 A_1(u,v)(t)$, $A_2(ru,rv)(t)\geq\varphi_2(t)A_2(u,v)(t)$.

再证 $A_1(h_1,h_2)\in P_{h_1}$, $A_2(h_1,h_2)\in P_{h_2}$, 设

$$
\begin{aligned}
r_1 = \min_{t\in[0,1]}\{f(t,l_1,l_2)\},\ R_1 = \max_{t\in[0,1]}\{f(t,L_1,L_2)\},\\
r_2 = \min_{t\in[0,1]}\{g(t,l_1,l_2)\},\ R_2 = \max_{t\in[0,1]}\{g(t,L_1,L_2)\},
\end{aligned}
$$

由 $(H_1),(H_2)$ 得

$$
\begin{aligned}
A_1(u,v)(t) &= \int_0^t (\int_s^1 f(\tau,u_1(\tau),v_1(\tau))d\tau)^{p-1}ds \\
&\geq \int_0^t (\int_s^1 f(\tau,l_1,l_2)d\tau)^{p-1}ds \\
&= r_1^{p-1}\int_0^t (1-s)^{p-1}ds \\
&= r_1^{p-1}h_1,
\end{aligned}
\tag{3.5}
$$

$$
\begin{aligned}
A_1(u,v)(t) &= \int_0^t (\int_s^1 f(\tau,u_1(\tau),v_1(\tau))d\tau)^{p-1}ds \\
&\leq \int_0^t (\int_s^1 f(\tau,L_1,L_2)d\tau)^{p-1}ds \\
&= R_1^{p-1}\int_0^t (1-s)^{p-1}ds \\
&= R_1^{p-1}h_1,
\end{aligned}
\tag{3.6}
$$

由 (3.5), (3.6) 知 $r_1^{p-1}h_1\leq A_1(u,v)(t)\leq R_1^{p-1}h_1$, 即 $A_1(u,v)\in P_{h_1}$. 同理可以得到 $A_2(u,v)\in P_{h_2}$.

最后由引理 2.6 可得如下结论:

(1) $\exists u_1,v_1\in P_{h_1}, u_2,v_2\in P_{h_2}, r\in(0,1)$, 使得 $r(v_1,v_2)\leq(u_1,u_2)\leq(v_1,v_2)$, 且

$$
u_1\leq A_1(u_1,v_1)\leq v_1,\ u_2\leq A_2(u_1,v_1)\leq v_2,
$$

$$
u_1(t)\leq\int_0^t (\int_s^1 f(\tau,u(\tau),v(\tau))d\tau)^{p-1}ds\leq v_1(t), t\in[0,1],
$$

$$u_2(t) \le \int_0^t (\int_s^1 g(\tau, u(\tau), v(\tau)) d\tau)^{p-1} ds \le v_2(t), t \in [0,1].$$

(2) 对任意固定的 λ , $\mu > 0$, 算子方程 $(u,v) = (\lambda^{p-1}A_1(u,v), \mu^{q-1}A_2(u,v))$ 有唯一的解 $(u^*_{\lambda,\mu}, v^*_{\lambda,\mu}) \in \bar{P}_h$, 使得 $(u^*_{\lambda,\mu}, v^*_{\lambda,\mu}) = A(u^*_{\lambda,\mu}, v^*_{\lambda,\mu})$. 因此系统 (1.6) 有唯一的正解 $(u^*_{\lambda,\mu}, v^*_{\lambda,\mu}) \in \bar{P}_h$.

(3) 对任意初始点 $(u_0, v_0) \in \bar{P}_h$, 定义

$$u_{n+1} = \lambda^{p-1}A_1(u_n, v_n)(t) = \lambda^{p-1}\int_0^t (\int_s^1 f(\tau, u_n(\tau), v_n(\tau)) d\tau)^{p-1} ds,\ n = 1, 2, ...$$

$$v_{n+1} = \mu^{q-1}A_2(u_n, v_n)(t) = \mu^{q-1}\int_0^t (\int_s^1 g(\tau, u_n(\tau), v_n(\tau)) d\tau)^{q-1} ds,\ n = 1, 2, ...$$

当 $n \to \infty$, $u_n(t) \to u^*_{\lambda,\mu}, v_n(t) \to v^*_{\lambda,\mu}$.

4. 举例

例 4.1. 考虑下面微分系统:

$$\begin{cases} -((u')^{p-1})' = 2(u^{\frac{1}{3}} + v^{\frac{1}{4}}) + 2a, t \in (0,1), \\ -((v')^{q-1})' = 3(u^{\frac{1}{5}} + v^{\frac{1}{6}}) + 3b \end{cases} \tag{4.1}$$

其中 a , $b > 0$, $0 < p-1 < 1$, $0 < q-1 < 1$, 设

$$f(t,u,v) = 2(u^{\frac{1}{3}} + v^{\frac{1}{4}}) + 2a,\ g(t,u,v) = 3(u^{\frac{1}{5}} + v^{\frac{1}{6}}) + 3b.$$

显然 $f, g : [0,1] \times [0,+\infty) \times [0,+\infty) \to [0,+\infty)$ 连续, 且 f, g 关于第二和第三变量是递增的. 令 $\varphi_1(r) = r^{\frac{1}{3}}$, $\varphi_2(r) = r^{\frac{1}{5}}$, $r \in (0,1)$, 易知 $\varphi_1(r) = r^{\frac{1}{3}} > r$, $\varphi_2(r) = r^{\frac{1}{5}} > r$. 对任意 $u, v \in R_+, t \in [0,1], r \in (0,1)$, 满足

$$\begin{aligned} f(t, ru, rv) &= 2((ru)^{\frac{1}{3}} + (rv)^{\frac{1}{4}}) + 2a \\ &\ge r^{\frac{1}{3}}[2(u^{\frac{1}{3}} + v^{\frac{1}{4}}) + 2a] \\ &\ge r^{\frac{1}{3(p-1)}}[2(u^{\frac{1}{3}} + v^{\frac{1}{4}}) + 2a] \\ &= \varphi_1^{p-1}(r) f(t,u,v), \end{aligned}$$

$$\begin{aligned} g(t, ru, rv) &= 3((ru)^{\frac{1}{5}} + (rv)^{\frac{1}{6}}) + 3b \\ &\ge r^{\frac{1}{5}}[3(u^{\frac{1}{5}} + v^{\frac{1}{6}}) + 3b] \\ &\ge r^{\frac{1}{5(q-1)}}[3(u^{\frac{1}{5}} + v^{\frac{1}{6}}) + 3b] \\ &= \varphi_2^{q-1}(r) f(t,u,v). \end{aligned}$$

另外

$$h_1(t) = \int_0^t (1-s)^{p-1} ds,\ h_2(t) = \int_0^t (1-s)^{q-1} ds,\ t \in [0,1],$$

$f(t,l_1,l_2) \geq f(t,0,0) = 2a > 0$, $g(t,l_1,l_2) \geq g(t,0,0) = 3b > 0$. 其中 l_1, l_2 由第三部分可知, 易知定理 3.1 中条件都满足. 故由定理 3.1 可得系统 4.1 有唯一的正解 $(u^*_{\lambda,\mu}, v^*_{\lambda,\mu}) \in \bar{P}_h$, 对任意初始点 $(u_0, v_0) \in \bar{P}_h$, 定义

$$u_{n+1} = \int_0^t (\int_s^1 [2(u_{n-1}^{\frac{1}{3}}(\tau) + v_{n-1}^{\frac{1}{4}}(\tau)) + 2a]d\tau)^{p-1} ds,\ n = 1, 2, ...$$

$$v_{n+1} = \int_0^t (\int_s^1 [3(u_{n-1}^{\frac{1}{5}}(\tau) + v_{n-1}^{\frac{1}{6}}(\tau)) + 3b]d\tau)^{q-1} ds,\ n = 1, 2, ...$$

当 $n \to \infty$, $u_n(t) \to u^*_{\lambda,\mu}$, $v_n(t) \to v^*_{\lambda,\mu}$.

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