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分数阶BAM模糊神经网络的全局Mittag-Leffler镇定

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摘　要

本文解决了分数阶BAM模糊神经网络的全局Mittag-Leffler (M-L)镇定问题。首先回顾了与分数阶微积分相关的基础知识，并建立了网络模型。其次，基于一种新的压缩映射和二范数分析方法严格证明了模型平衡点的存在唯一性。最后，通过设计一种简洁有效的线性控制器导出了分数阶BAM模糊神经网络实现全局M-L镇定的充分性判据。

关键词

BAM神经网络，Mittag-Leffler镇定，分数阶，模糊逻辑

Global Mittag-Leffler Stabilization of BAM Fuzzy Neural Networks with Fractional-Order

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Abstract

This paper deals with the issue of global Mittag-Leffler (M-L) stabilization for fractional-order

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BAM fuzzy neural networks (FBAMFNNs). Firstly, some necessary knowledge related to fractional calculus are reviewed, and the model of FBAMFNN is established. Next, the existence and uniqueness of equilibrium point is proved based on constructing a novel contraction mapping and 2-norm analysis method. Finally, the sufficient criterion is derived to realize global M-L stabilization of FBAMFNNs by designing a concise and effective linear controller.

Keywords

BAM Neural Networks, Mittag-Leffler Stabilization, Fractional-Order, Fuzzy Logic

1. 引言

在过去的数十年中，神经网络因在信号处理、保密通信和模式识别等领域的广泛应用激发了许多国内外研究者的兴趣[1] [2]。作为单层到双层模式匹配电路的推广，BAM 神经网络最早由 Kosko 于 1987 年创立[3]，其在自动控制与电力系统等方面具有重要应用前景[4]，近年来 BAM 神经网络的动力学分析受到了学者们的广泛关注，并取得了许多有价值的成果[5] [6]。

众所周知，不确定性或模糊性在现实中是不可避免的，模糊逻辑是解决上述问题的一种重要工具，通过考虑模糊因素，并将模糊 AND 与模糊 OR 运算融入经典的神经网络模型，学者们提出并陆续探究了各类模糊神经网络的动力学行为[7] [8]。文献[8]采用非抖振量化控制研究了具有时滞和脉冲效应的不连续非恒等模糊 BAM 神经网络的有限时间同步。分数阶微积分不仅是整数阶微积分在阶数意义上的推广，与整数阶微积分相比，分数阶微积分在生物数学和工程控制等实际应用中具有更强的建模能力。无限记忆性与遗传性作为分数阶微积分的两大独特优势，将分数阶微积分与神经网络相结合有助于更精确地刻画神经网络的动力学行为。分数阶模糊神经网络作为一种重要的网络模型，得到了许多研究者的关注[9] [10]。文献[9]基于非线性反馈控制考虑了分数阶模糊神经网络的完全同步与有限时间同步。文献[10]采用直接四元数法分析了分数阶四元值模糊 BAM 神经网络的有限时间镇定。

镇定性在控制工程与系统辨识等实际应用中至关重要，设计合适的控制策略是实现系统镇定的关键。迄今为止，反馈控制、间接控制、事件触发控制和混合控制等多种控制策略被陆续提出并用于神经网络的镇定性研究[11] [12]，值得注意的是与其它控制策略相比，反馈控制更加简洁有效，然而目前很少有研究基于反馈控制策略分析分数阶 BAM 模糊神经网络的全局 M-L 镇定性问题，这激发了我们进一步研究的兴趣。

受上述分析的启发，本文将探究分数阶 BAM 模糊神经网络的全局 M-L 镇定问题。本文的创新点可归纳为以下三方面。首先，构建了分数阶 BAM 模糊神经网络模型。其次，基于压缩映射原理证明了分数阶 BAM 模糊神经网络平衡点的存在唯一性。最后，设计了一种简洁有效的线性反馈控制器，结合不等式分析技巧得到了分数阶 BAM 模糊神经网络实现全局 M-L 镇定的充分性判据。

本文结构安排如下：第二节给出了分数阶 BAM 模糊神经网络模型，回顾了分数阶微积分的相关定义与引理，为后文研究需要对激活函数做出来假设。第 3 节证明了分数阶 BAM 模糊神经网络平衡点的存在唯一性。第 4 节分析了分数阶 BAM 模糊神经网络的全局 M-L 镇定问题。第 5 节给出了总结与展望。

符号：R 代表实数集，R^{+} 表示正实值集，$N=\{1,2,\cdots,n\},M=\{1,2,\cdots,m\}$，对任意 n 维实值向量

$\tau = (\tau_1, \tau_2, \cdots, \tau_n)^{\mathrm{T}} \in R^n$，$\tau$ 的 2-范数定义为 $\|\tau\| = \sqrt{\sum_{\iota=1}^{n} |\tau_\iota|^2}$。

2. 预备知识与模型描述

定义 1. [13]阶数为 $0<\nu<1$ 的函数 $\omega(t)$ 的 Caputo 分数阶导数定义为

$$ {}_{t_0}^{c}D_t^{\nu}\omega(t) = \frac{1}{\Gamma(1-\nu)}\int_{t_0}^{t}\frac{\omega'(s)}{(t-s)^{\nu}}\mathrm{d}s, $$

其中 $\Gamma(\nu) = \int_{t_0}^{+\infty} \mathrm{e}^{-t}t^{\nu-1}\mathrm{d}t$ 为 Gamma 函数。

考虑如下分数阶 BAM 模糊神经网络模型：

$$ \begin{cases} {}_{t_0}^{c}D_t^{\nu}\alpha_\iota(t) = -\theta_\iota\alpha_\iota(t) + \sum_{\kappa=1}^{m}\mu_{\iota\kappa}\omega_\kappa(\beta_\kappa(t)) + \underset{\kappa=1}{\overset{m}{\vee}}\phi_{\iota\kappa}\omega_\kappa(\beta_\kappa(t)) + \underset{\kappa=1}{\overset{m}{\wedge}}\psi_{\iota\kappa}\omega_\kappa(\beta_\kappa(t)) + \Theta_\iota(t), \\ {}_{t_0}^{c}D_t^{\nu}\beta_\kappa(t) = -\vartheta_\kappa\beta_\kappa(t) + \sum_{\iota=1}^{n}\zeta_{\kappa\iota}\varpi_\iota(\alpha_\iota(t)) + \underset{\iota=1}{\overset{n}{\vee}}\delta_{\kappa\iota}\varpi_\iota(\alpha_\iota(t)) + \underset{\iota=1}{\overset{n}{\wedge}}\eta_{\kappa\iota}\varpi_\iota(\alpha_\iota(t)) + \Lambda_\kappa(t), \end{cases} \tag{1} $$

其中 $\iota\in N, \kappa\in M$，$\alpha_\iota(t)$ 与 $\beta_\kappa(t)$ 分别代表第 ι 个和第 κ 个神经元的状态。n 与 m 依次表示第一层和第二层中神经元的数量。θ_ι 和 ϑ_κ 是第 ι 个和第 κ 个神经元的衰减系数，$\mu_{\iota\kappa}$ 与 $\zeta_{\kappa\iota}$ 为连接权重，$\phi_{\iota\kappa}$ 和 $\delta_{\kappa\iota}$ 表示模糊反馈最大模板的连接权重，$\psi_{\iota\kappa}$ 和 $\eta_{\kappa\iota}$ 代表模糊反馈最小模板的连接权重，$\omega_\kappa(t)$ 与 $\varpi_\iota(t)$ 表示第 κ 个和第 ι 个神经元的激活函数。$\wedge$ 和 $\vee$ 代表模糊 OR 与 AND 运算。$\Theta_\iota(t)$ 与 $\Lambda_\kappa(t)$ 分别表示不同层中的外部输入。

为便于本文后续研究，对上述激活函数作出如下假设：

假设 1. [8]对任意 α,β，存在正常数 λ_κ 与 χ_ι 使得

$$ |\omega_\kappa(\alpha) - \omega_\kappa(\beta)| \le \lambda_\kappa|\alpha-\beta|, $$
$$ |\varpi_\iota(\alpha) - \varpi_\iota(\beta)| \le \chi_\iota|\alpha-\beta|. $$

引理 1. [7]若 $\alpha_\iota,\tilde{\alpha}_\iota$ 与 $\beta_\kappa,\tilde{\beta}_\kappa$ 分别为模型(1)的状态，则模型(1)中的激活函数满足下列不等式

$$ \left|\underset{\kappa=1}{\overset{m}{\vee}}\phi_{\iota\kappa}\omega_\kappa(\beta_\kappa(t)) - \underset{\kappa=1}{\overset{m}{\vee}}\phi_{\iota\kappa}\omega_\kappa(\tilde{\beta}_\kappa(t))\right| \le \sum_{\kappa=1}^{m}|\phi_{\iota\kappa}|\left|\omega_\kappa(\beta_\kappa(t)) - \omega_\kappa(\tilde{\beta}_\kappa(t))\right|, $$

$$ \left|\underset{\kappa=1}{\overset{m}{\wedge}}\psi_{\iota\kappa}\omega_\kappa(\beta_\kappa(t)) - \underset{\kappa=1}{\overset{m}{\wedge}}\psi_{\iota\kappa}\omega_\kappa(\tilde{\beta}_\kappa(t))\right| \le \sum_{\kappa=1}^{m}|\psi_{\iota\kappa}|\left|\omega_\kappa(\beta_\kappa(t)) - \omega_\kappa(\tilde{\beta}_\kappa(t))\right|, $$

$$ \left|\underset{\iota=1}{\overset{n}{\vee}}\delta_{\iota\kappa}\varpi_\iota(\alpha_\iota(t)) - \underset{\iota=1}{\overset{n}{\vee}}\delta_{\iota\kappa}\varpi_\iota(\tilde{\alpha}_\iota(t))\right| \le \sum_{\iota=1}^{n}|\delta_{\iota\kappa}|\left|\varpi_\iota(\alpha_\iota(t)) - \varpi_\iota(\tilde{\alpha}_\iota(t))\right|, $$

$$ \left|\underset{\iota=1}{\overset{n}{\wedge}}\eta_{\iota\kappa}\varpi_\iota(\alpha_\iota(t)) - \underset{\iota=1}{\overset{n}{\wedge}}\eta_{\iota\kappa}\varpi_\iota(\tilde{\alpha}_\iota(t))\right| \le \sum_{\iota=1}^{n}|\eta_{\iota\kappa}|\left|\varpi_\iota(\alpha_\iota(t)) - \varpi_\iota(\tilde{\alpha}_\iota(t))\right|. $$

引理 2. [14]若 $\sigma=(\sigma_1,\sigma_2,\cdots,\sigma_m)$ 与 $\pi=(\pi_1,\pi_2,\cdots,\pi_m)$ 是成比例的序列，则有

$$ \left|\sum_{\kappa=1}^{m}\sigma_\kappa\pi_\kappa\right|^2 \le \left(\sum_{\kappa=1}^{m}|\sigma_\kappa|^2\right)\left(\sum_{\kappa=1}^{m}|\pi_\kappa|^2\right). $$

引理 3. [15]若 $V(t)$ 为定义在 $[t_0,a)$ 上的连续可微函数，则对任意的常数 b 与 $0<\nu<1$ 有

$$ {}_{t_0}^{c}D_t^{\nu}(V(t)-b)^2 \le 2(V(t)-b)\,{}_{t_0}^{c}D_t^{\nu}V(t). $$

引理 4. [16]如果 $V(t)$ 是定义在 $[t_0,+\infty)$ 上的非负连续函数，并且对任意初始时刻 t_0 以及常数 $0<\nu<1, \Omega\in R$ 满足

$$ {}_{t_0}^{c}D_t^{\nu}V(t) \le -\Omega V(t), $$

那么有 $V(t) \le V(t_0)E_\nu\left(-\Omega(t-t_0)^\nu\right)$。

定义 2. 如果存在 $\varsigma = \left(\alpha_1^*,\alpha_2^*,\cdots,\alpha_n^*,\beta_1^*,\beta_2^*,\cdots,\beta_n^*\right)^{\mathrm{T}}$ 使得

$$ \begin{cases} 0 = -\theta_\iota\alpha_\iota^*(t) + \sum_{\kappa=1}^{m}\mu_{\iota\kappa}\omega_\kappa\left(\beta_\kappa^*(t)\right) + \bigvee_{\kappa=1}^{m}\phi_{\iota\kappa}\omega_\kappa\left(\beta_\kappa^*(t)\right) + \bigwedge_{\kappa=1}^{m}\psi_{\iota\kappa}\omega_\kappa\left(\beta_\kappa^*(t)\right) + \Theta_\iota(t), \\ 0 = -\vartheta_\kappa\beta_\kappa^*(t) + \sum_{\iota=1}^{n}\zeta_{\kappa\iota}\varpi_\iota\left(\alpha_\iota^*(t)\right) + \bigvee_{\iota=1}^{n}\delta_{\kappa\iota}\varpi_\iota\left(\alpha_\iota^*(t)\right) + \bigwedge_{\iota=1}^{n}\eta_{\kappa\iota}\varpi_\iota\left(\alpha_\iota^*(t)\right) + \Lambda_\kappa(t), \end{cases} $$

那么 ς 是分数阶 BAM 模糊神经网络(1)的平衡点。

定义 3. 如果存在正常数 h,Ω,l，使得对系统(1)的任意解 $\xi = \left(\alpha_1,\alpha_2,\cdots,\alpha_n,\beta_1,\beta_2,\cdots,\beta_n\right)^{\mathrm{T}}$ 与 $t \ge t_0$ 有

$$ \left\|\xi - \xi^*\right\| \le \left(h\left\|\xi(t_0) - \xi^*\right\|E_\nu\left(-\Omega(t-t_0)^\nu\right)\right)^{\frac{1}{l}}, $$

那么称系统(1)在平衡点 $\xi^* = \left(\alpha_1^*,\alpha_2^*,\cdots,\alpha_n^*,\beta_1^*,\beta_2^*,\cdots,\beta_n^*\right)^{\mathrm{T}}$ 处是全局 M-L 镇定的，其中 $\xi(t_0)$ 为系统(1)的初始值。

3. 平衡点的存在唯一性

在本节中，通过构造新的压缩映射并结合二范数分析方法严格证明了分数阶 BAM 模糊神经网络平衡点的存在唯一性。

定理 1. 在假设 1 下，如果满足下列条件

$$ 0 < \max\left\{\max_{1\le\iota\le n}\left\{\sum_{\kappa=1}^{m}\frac{\lambda_\kappa\left(\left|\mu_{\iota\kappa}\right| + \left|\phi_{\iota\kappa}\right| + \left|\psi_{\iota\kappa}\right|\right)}{\vartheta_\kappa}\right\}, \max_{1\le\kappa\le m}\left\{\sum_{\iota=1}^{n}\frac{\chi_\iota\left(\left|\zeta_{\kappa\iota}\right| + \left|\delta_{\kappa\iota}\right| + \left|\eta_{\kappa\iota}\right|\right)}{\theta_\iota}\right\}\right\} < 1, \tag{2} $$

则分数阶 BAM 模糊神经网络(1)存在唯一的平衡点 $\varsigma = \left(\alpha_1^*,\alpha_2^*,\cdots,\alpha_n^*,\beta_1^*,\beta_2^*,\cdots,\beta_n^*\right)^{\mathrm{T}}$。

证明：记 $\rho_\iota^* = \theta_\iota\alpha_\iota^*, \gamma_\kappa^* = \vartheta_\kappa\beta_\kappa^*$，构造如下映射

$$ \Pi_\iota(\rho,\gamma) = \sum_{\kappa=1}^{m}\mu_{\iota\kappa}\omega_\kappa\left(\frac{\gamma_\kappa}{\vartheta_\kappa}\right) + \bigvee_{\kappa=1}^{m}\phi_{\iota\kappa}\omega_\kappa\left(\frac{\gamma_\kappa}{\vartheta_\kappa}\right) + \bigwedge_{\kappa=1}^{m}\psi_{\iota\kappa}\omega_\kappa\left(\frac{\gamma_\kappa}{\vartheta_\kappa}\right) + \Theta_\iota(t), $$

$$ \Xi_\kappa(\rho,\gamma) = \sum_{\iota=1}^{n}\zeta_{\kappa\iota}\varpi_\iota\left(\frac{\rho_\iota}{\theta_\iota}\right) + \bigvee_{\iota=1}^{n}\delta_{\kappa\iota}\varpi_\iota\left(\frac{\rho_\iota}{\theta_\iota}\right) + \bigwedge_{\iota=1}^{n}\eta_{\kappa\iota}\varpi_\iota\left(\frac{\rho_\iota}{\theta_\iota}\right) + \Lambda_\kappa(t), $$

其中 $\Pi(\rho,\gamma) = \left(\Pi_1(\rho,\gamma),\cdots,\Pi_n(\rho,\gamma)\right)^{\mathrm{T}}$，$\Xi(\rho,\gamma) = \left(\Xi_1(\rho,\gamma),\cdots,\Xi_n(\rho,\gamma)\right)^{\mathrm{T}}$，$(\rho,\gamma) = \left((\rho_1,\gamma_1),\cdots,(\rho_m,\gamma_m)\right)^{\mathrm{T}}$ 并且 $m \ge n$，接下来证明 (Π,Ξ) 为一压缩映射。

对任意的 (ρ,γ) 与 $(\tilde{\rho},\tilde{\gamma})$，基于假设 1，引理 1 和 2 可得

$$ \begin{aligned} &\left\|(\Pi,\Xi)(\rho,\gamma) - (\Pi,\Xi)(\tilde{\rho},\tilde{\gamma})\right\| \\ &= \left\|\Pi(\rho,\gamma) - \Pi(\tilde{\rho},\tilde{\gamma})\right\| + \left\|\Xi(\rho,\gamma) - \Xi(\tilde{\rho},\tilde{\gamma})\right\| \\ &= \left(\sum_{\iota=1}^{n}\left|\sum_{\kappa=1}^{m}\mu_{\iota\kappa}\omega_\kappa\left(\frac{\gamma_\kappa}{\vartheta_\kappa}\right) - \sum_{\kappa=1}^{m}\mu_{\iota\kappa}\omega_\kappa\left(\frac{\tilde{\gamma}_\kappa}{\vartheta_\kappa}\right) + \bigvee_{\kappa=1}^{m}\phi_{\iota\kappa}\omega_\kappa\left(\frac{\gamma_\kappa}{\vartheta_\kappa}\right)\right.\right. \\ &\quad \left.\left. - \bigvee_{\kappa=1}^{m}\phi_{\iota\kappa}\omega_\kappa\left(\frac{\tilde{\gamma}_\kappa}{\vartheta_\kappa}\right) + \bigwedge_{\kappa=1}^{m}\psi_{\iota\kappa}\omega_\kappa\left(\frac{\gamma_\kappa}{\vartheta_\kappa}\right) - \bigwedge_{\kappa=1}^{m}\psi_{\iota\kappa}\omega_\kappa\left(\frac{\tilde{\gamma}_\kappa}{\vartheta_\kappa}\right)\right|^2\right)^{\frac{1}{2}} \\ &\quad + \left(\sum_{\kappa=1}^{m}\left|\sum_{\iota=1}^{n}\zeta_{\kappa\iota}\varpi_\iota\left(\frac{\rho_\iota}{\theta_\iota}\right) - \sum_{\iota=1}^{n}\zeta_{\kappa\iota}\varpi_\iota\left(\frac{\tilde{\rho}_\iota}{\theta_\iota}\right) + \bigvee_{\iota=1}^{n}\delta_{\kappa\iota}\varpi_\iota\left(\frac{\rho_\iota}{\theta_\iota}\right)\right.\right. \end{aligned} $$

$$
\begin{aligned}
&-\mathop{\vee}_{\iota=1}^{n}\delta_{\kappa\iota}\varpi_{\iota}\left(\frac{\tilde{\rho}_{\iota}}{\theta_{\iota}}\right)+\mathop{\wedge}_{\iota=1}^{n}\eta_{\kappa\iota}\varpi_{\iota}\left(\frac{\rho_{\iota}}{\theta_{\iota}}\right)-\mathop{\wedge}_{\iota=1}^{n}\eta_{\kappa\iota}\varpi_{\iota}\left(\frac{\tilde{\rho}_{\iota}}{\theta_{\iota}}\right)\Bigg|^{2}\Bigg)^{\frac{1}{2}}\\
\le&\left(\sum_{\iota=1}^{n}\left|\sum_{\kappa=1}^{m}\left|\mu_{\iota\kappa}\right|\left|\omega_{\kappa}\left(\frac{\gamma_{\kappa}}{\vartheta_{\kappa}}\right)-\omega_{\kappa}\left(\frac{\tilde{\gamma}_{\kappa}}{\vartheta_{\kappa}}\right)\right|+\mathop{\vee}_{\kappa=1}^{m}\left|\phi_{\iota\kappa}\right|\left|\omega_{\kappa}\left(\frac{\gamma_{\kappa}}{\vartheta_{\kappa}}\right)-\omega_{\kappa}\left(\frac{\tilde{\gamma}_{\kappa}}{\vartheta_{\kappa}}\right)\right|\right.\right.\\
&\left.\left.+\mathop{\wedge}_{\kappa=1}^{m}\left|\psi_{\iota\kappa}\right|\left|\omega_{\kappa}\left(\frac{\gamma_{\kappa}}{\vartheta_{\kappa}}\right)-\omega_{\kappa}\left(\frac{\tilde{\gamma}_{\kappa}}{\vartheta_{\kappa}}\right)\right|\right|^{2}\right)^{\frac{1}{2}}+\left(\sum_{\kappa=1}^{m}\left|\sum_{\iota=1}^{n}\left|\zeta_{\kappa\iota}\right|\left|\varpi_{\iota}\left(\frac{\rho_{\iota}}{\theta_{\iota}}\right)-\varpi_{\iota}\left(\frac{\tilde{\rho}_{\iota}}{\theta_{\iota}}\right)\right|\right.\right.\\
&\left.\left.+\mathop{\vee}_{\iota=1}^{n}\left|\delta_{\kappa\iota}\right|\left|\varpi_{\iota}\left(\frac{\rho_{\iota}}{\theta_{\iota}}\right)-\varpi_{\iota}\left(\frac{\tilde{\rho}_{\iota}}{\theta_{\iota}}\right)\right|+\mathop{\wedge}_{\iota=1}^{n}\left|\eta_{\kappa\iota}\right|\left|\varpi_{\iota}\left(\frac{\rho_{\iota}}{\theta_{\iota}}\right)-\varpi_{\iota}\left(\frac{\tilde{\rho}_{\iota}}{\theta_{\iota}}\right)\right|\right|^{2}\right)^{\frac{1}{2}}\\
\le&\left(\sum_{\iota=1}^{n}\left(\sum_{\kappa=1}^{m}\frac{\lambda_{\kappa}\left(\left|\mu_{\iota\kappa}\right|+\left|\phi_{\iota\kappa}\right|+\left|\psi_{\iota\kappa}\right|\right)}{\vartheta_{\kappa}}\left|\gamma_{\kappa}-\tilde{\gamma}_{\kappa}\right|\right)^{2}\right)^{\frac{1}{2}}\\
&+\left(\sum_{\kappa=1}^{m}\left(\sum_{\iota=1}^{n}\frac{\chi_{\iota}\left(\left|\zeta_{\kappa\iota}\right|+\left|\delta_{\kappa\iota}\right|+\left|\eta_{\kappa\iota}\right|\right)}{\theta_{\iota}}\left|\rho_{\iota}-\tilde{\rho}_{\iota}\right|\right)^{2}\right)^{\frac{1}{2}}\\
\le&\left(\max_{1\le\iota\le n}\left\{\sum_{\kappa=1}^{m}\frac{\lambda_{\kappa}\left(\left|\mu_{\iota\kappa}\right|+\left|\phi_{\iota\kappa}\right|+\left|\psi_{\iota\kappa}\right|\right)}{\vartheta_{\kappa}}\right\}^{2}\sum_{\kappa=1}^{m}\left|\gamma_{\kappa}-\tilde{\gamma}_{\kappa}\right|^{2}\right)^{\frac{1}{2}}\\
&+\left(\max_{1\le\kappa\le m}\left\{\sum_{\iota=1}^{n}\frac{\chi_{\iota}\left(\left|\zeta_{\kappa\iota}\right|+\left|\delta_{\kappa\iota}\right|+\left|\eta_{\kappa\iota}\right|\right)}{\theta_{\iota}}\right\}^{2}\sum_{\iota=1}^{n}\left|\rho_{\iota}-\tilde{\rho}_{\iota}\right|^{2}\right)^{\frac{1}{2}}\\
\le&\max_{1\le\iota\le n}\left\{\sum_{\kappa=1}^{m}\frac{\lambda_{\kappa}\left(\left|\mu_{\iota\kappa}\right|+\left|\phi_{\iota\kappa}\right|+\left|\psi_{\iota\kappa}\right|\right)}{\vartheta_{\kappa}}\right\}\left\|\gamma-\tilde{\gamma}\right\|\\
&+\max_{1\le\kappa\le m}\left\{\sum_{\iota=1}^{n}\frac{\chi_{\iota}\left(\left|\zeta_{\kappa\iota}\right|+\left|\delta_{\kappa\iota}\right|+\left|\eta_{\kappa\iota}\right|\right)}{\theta_{\iota}}\right\}\left\|\rho-\tilde{\rho}\right\|.
\end{aligned}
$$

结合条件(2)与上式，我们有

$$\left\|(\Pi,\Xi)(\rho,\gamma)-(\Pi,\Xi)(\tilde{\rho},\tilde{\gamma})\right\|_{2}\le\left\|\rho-\tilde{\rho}\right\|_{2}+\left\|\gamma-\tilde{\gamma}\right\|_{2}=\left\|(\rho,\gamma)-(\tilde{\rho},\tilde{\gamma})\right\|_{2}.$$

故(Π,Ξ)是一个压缩映射，从而存在唯一的不动点(ρ^{*},γ^{*})使得$(\Pi,\Xi)(\rho^{*},\gamma^{*})=(\rho^{*},\gamma^{*})$，即

$$
\begin{aligned}
\rho_{\iota}^{*}&=\sum_{\kappa=1}^{m}\mu_{\iota\kappa}\omega_{\kappa}\left(\frac{\gamma_{\kappa}^{*}}{\vartheta_{\kappa}}\right)+\mathop{\vee}_{\kappa=1}^{m}\phi_{\iota\kappa}\omega_{\kappa}\left(\frac{\gamma_{\kappa}^{*}}{\vartheta_{\kappa}}\right)+\mathop{\wedge}_{\kappa=1}^{m}\psi_{\iota\kappa}\omega_{\kappa}\left(\frac{\gamma_{\kappa}^{*}}{\vartheta_{\kappa}}\right)+\Theta_{\iota}(t),\\
\gamma_{\kappa}^{*}&=\sum_{\iota=1}^{n}\zeta_{\kappa\iota}\varpi_{\iota}\left(\frac{\rho_{\iota}^{*}}{\theta_{\iota}}\right)+\mathop{\vee}_{\iota=1}^{n}\delta_{\kappa\iota}\varpi_{\iota}\left(\frac{\rho_{\iota}^{*}}{\theta_{\iota}}\right)+\mathop{\wedge}_{\iota=1}^{n}\eta_{\kappa\iota}\varpi_{\iota}\left(\frac{\rho_{\iota}^{*}}{\theta_{\iota}}\right)+\Lambda_{\kappa}(t).
\end{aligned}
\tag{3}
$$

式等价于

$$
\begin{cases}
0=-\theta_{\iota}\alpha_{\iota}^{*}(t)+\sum_{\kappa=1}^{m}\mu_{\iota\kappa}\omega_{\kappa}\left(\beta_{\kappa}^{*}(t)\right)+\mathop{\vee}_{\kappa=1}^{m}\phi_{\iota\kappa}\omega_{\kappa}\left(\beta_{\kappa}^{*}(t)\right)+\mathop{\wedge}_{\kappa=1}^{m}\psi_{\iota\kappa}\omega_{\kappa}\left(\beta_{\kappa}^{*}(t)\right)+\Theta_{\iota}(t),\\
0=-\vartheta_{\kappa}\beta_{\kappa}^{*}(t)+\sum_{\iota=1}^{n}\zeta_{\kappa\iota}\varpi_{\iota}\left(\alpha_{\iota}^{*}(t)\right)+\mathop{\vee}_{\iota=1}^{n}\delta_{\kappa\iota}\varpi_{\iota}\left(\alpha_{\iota}^{*}(t)\right)+\mathop{\wedge}_{\iota=1}^{n}\eta_{\kappa\iota}\varpi_{\iota}\left(\alpha_{\iota}^{*}(t)\right)+\Lambda_{\kappa}(t).
\end{cases}
$$

由定义 2 可知，分数阶 BAM 模糊神经网络(1)有唯一的平衡点 $\varsigma=\left(\alpha_1^*,\alpha_2^*,\cdots,\alpha_n^*,\beta_1^*,\beta_2^*,\cdots,\beta_n^*\right)^{\mathrm{T}}$。

注 1. 通过将分数阶导数、模糊逻辑等因素考虑在内，分数阶 BAM 模糊神经网络模型比分数阶神经网络[12] [15]、模糊神经网络[7] [11]、BAM 神经网络[4] [5]等更加一般化，并且实用性更广。

4. 全局 M-L 镇定性

本节设计了一种简洁有效的线性反馈控制器，基于分数阶理论与不等式分析技巧，我们得到了系统(1)实现全局 M-L 镇定的充分性判据。

接下来为将系统(1)的平衡点转换到原点，作变换 $r_\iota(t)=\alpha_\iota(t)-\alpha_\iota^*,\hbar_\kappa(t)=\hbar_\kappa(t)-\hbar_\kappa^*$，从而系统(1)转换后的形式为

$$\begin{cases}{}_{t_0}^{c}D_t^{\nu}r_\iota(t)=-\theta_\iota r_\iota(t)+\sum\limits_{\kappa=1}^{m}\mu_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right)+\mathop{\vee}\limits_{\kappa=1}^{m}\phi_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right)\\ \qquad +\mathop{\wedge}\limits_{\kappa=1}^{m}\psi_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right),\\ {}_{t_0}^{c}D_t^{\nu}\hbar_\kappa(t)=-\vartheta_\kappa\hbar_\kappa(t)+\sum\limits_{\iota=1}^{n}\zeta_{\kappa\iota}\left(\varpi_\iota\left(\alpha_\iota(t)\right)-\varpi_\iota\left(\alpha_\iota^*\right)\right)+\mathop{\vee}\limits_{\iota=1}^{n}\delta_{\kappa\iota}\left(\varpi_\iota\left(\alpha_\iota(t)\right)-\varpi_\iota\left(\alpha_\iota^*\right)\right)\\ \qquad +\mathop{\wedge}\limits_{\iota=1}^{n}\eta_{\kappa\iota}\left(\varpi_\iota\left(\alpha_\iota(t)\right)-\varpi_\iota\left(\alpha_\iota^*\right)\right),\end{cases}\tag{4}$$

(4)的受控形式为

$$\begin{cases}{}_{t_0}^{c}D_t^{\nu}r_\iota(t)=-\theta_\iota r_\iota(t)+\sum\limits_{\kappa=1}^{m}\mu_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right)+\mathop{\vee}\limits_{\kappa=1}^{m}\phi_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right)\\ \qquad +\mathop{\wedge}\limits_{\kappa=1}^{m}\psi_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right)+\hat{u}_\iota(t),\\ {}_{t_0}^{c}D_t^{\nu}\hbar_\kappa(t)=-\vartheta_\kappa\hbar_\kappa(t)+\sum\limits_{\iota=1}^{n}\zeta_{\kappa\iota}\left(\varpi_\iota\left(\alpha_\iota(t)\right)-\varpi_\iota\left(\alpha_\iota^*\right)\right)+\mathop{\vee}\limits_{\iota=1}^{n}\delta_{\kappa\iota}\left(\varpi_\iota\left(\alpha_\iota(t)\right)-\varpi_\iota\left(\alpha_\iota^*\right)\right)\\ \qquad +\mathop{\wedge}\limits_{\iota=1}^{n}\eta_{\kappa\iota}\left(\varpi_\iota\left(\alpha_\iota(t)\right)-\varpi_\iota\left(\alpha_\iota^*\right)\right)+\breve{u}_\iota(t),\end{cases}\tag{5}$$

其中 $\hat{u}_\iota(t)$ 与 $\breve{u}_\kappa(t)$ 为如下所设计的线性反馈控制器

$$\begin{cases}\hat{u}_\iota(t)=-\ell_\iota r_\iota(t),\\ \breve{u}_\iota(t)=-\wp_\kappa\hbar_\kappa(t),\end{cases}\tag{6}$$

$$\ell_\iota(t),\wp_\kappa(t)\in R^+.$$

定理 2. 基于假设 1 和控制器(6)，分数阶 BAM 模糊神经网络(1)在平衡点处是全局 M-L 镇定的。

证明：构造 Lyapunov 函数如下

$$V(t)=\frac{1}{2}\left[\sum_{\iota=1}^{n}r_\iota^2(t)+\sum_{\kappa=1}^{m}\hbar_\kappa^2(t)\right].$$

根据引理 3，求 $V(t)$ 沿系统(5)在控制器(6)下的 Caputo 分数阶导数可得

$$\begin{aligned}{}_{t_0}^{c}D_t^{\nu}V(t)&\le\sum_{\iota=1}^{n}r_\iota(t)\,{}_{t_0}^{c}D_t^{\nu}r_\iota(t)+\sum_{\kappa=1}^{m}\hbar_\kappa(t)\,{}_{t_0}^{c}D_t^{\nu}\hbar_\kappa(t)\\ &=\sum_{\iota=1}^{n}r_\iota(t)\Bigg[-\theta_\iota r_\iota(t)+\sum_{\kappa=1}^{m}\mu_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right)\\ &\quad+\mathop{\vee}_{\kappa=1}^{m}\phi_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right)+\mathop{\wedge}_{\kappa=1}^{m}\psi_{\iota\kappa}\left(\omega_\kappa\left(\beta_\kappa(t)\right)-\omega_\kappa\left(\beta_\kappa^*\right)\right)-\ell_\iota r_\iota(t)\Bigg]\end{aligned}$$

$$
\begin{aligned}
&+\sum_{\kappa=1}^{m}\hbar_{\kappa}(t)\Big[-\vartheta_{\kappa}\hbar_{\kappa}(t)+\sum_{\iota=1}^{n}\zeta_{\kappa\iota}\left(\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right)\\
&+\bigvee_{\iota=1}^{n}\delta_{\kappa\iota}\left(\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right)+\bigwedge_{\iota=1}^{n}\eta_{\kappa\iota}\left(\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right)-\wp_{\kappa}\hbar_{\kappa}(t)\Big]\\
&\le-\sum_{\iota=1}^{n}(\theta_{\iota}+\ell_{\iota})r_{\iota}^{2}(t)-\sum_{\kappa=1}^{m}(\vartheta_{\kappa}+\wp_{\kappa})\hbar_{\kappa}^{2}(t)+\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}|\mu_{\iota\kappa}||r_{\iota}(t)|\left|\omega_{\kappa}\left(\beta_{\kappa}(t)\right)-\omega_{\kappa}\left(\beta_{\kappa}^{*}\right)\right|\\
&+\sum_{\iota=1}^{n}|r_{\iota}(t)|\left|\bigvee_{\kappa=1}^{m}\phi_{\iota\kappa}\left(\omega_{\kappa}\left(\beta_{\kappa}(t)\right)-\omega_{\kappa}\left(\beta_{\kappa}^{*}\right)\right)\right|+\sum_{\iota=1}^{n}|r_{\iota}(t)|\left|\bigwedge_{\kappa=1}^{m}\psi_{\iota\kappa}\left(\omega_{\kappa}\left(\beta_{\kappa}(t)\right)-\omega_{\kappa}\left(\beta_{\kappa}^{*}\right)\right)\right|\\
&+\sum_{\kappa=1}^{m}|\zeta_{\kappa\iota}||\hbar_{\kappa}(t)|\left|\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right|+\sum_{\kappa=1}^{m}|\hbar_{\kappa}(t)|\left|\bigvee_{\iota=1}^{n}\delta_{\kappa\iota}\left(\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right)\right|\\
&+\sum_{\kappa=1}^{m}|\hbar_{\kappa}(t)|\left|\bigwedge_{\iota=1}^{n}\eta_{\kappa\iota}\left(\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right)\right|.
\end{aligned}
\tag{7}
$$

由假设 1 可知

$$
\begin{aligned}
&\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}|\mu_{\iota\kappa}||r_{\iota}(t)|\left|\omega_{\kappa}\left(\beta_{\kappa}(t)\right)-\omega_{\kappa}\left(\beta_{\kappa}^{*}\right)\right|\\
&\le\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\lambda_{\kappa}|\mu_{\iota\kappa}||r_{\iota}(t)||\hbar_{\kappa}(t)|\\
&\le\frac{1}{2}\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\lambda_{\kappa}|\mu_{\iota\kappa}|\left(r_{\iota}^{2}(t)+\hbar_{\kappa}^{2}(t)\right),
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&\sum_{\kappa=1}^{m}\sum_{\iota=1}^{n}|\zeta_{\kappa\iota}||\hbar_{\kappa}(t)|\left|\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right|\\
&\le\sum_{\kappa=1}^{m}\sum_{\iota=1}^{n}\chi_{\iota}|\zeta_{\kappa\iota}||\hbar_{\kappa}(t)||r_{\iota}(t)|\\
&\le\frac{1}{2}\sum_{\kappa=1}^{m}\sum_{\iota=1}^{n}\chi_{\iota}|\zeta_{\kappa\iota}|\left(\hbar_{\kappa}^{2}(t)+r_{\iota}^{2}(t)\right).
\end{aligned}
\tag{9}
$$

根据引理 1 和假设 1 有

$$
\begin{aligned}
&\sum_{\iota=1}^{n}|r_{\iota}(t)|\left|\bigvee_{\kappa=1}^{m}\phi_{\iota\kappa}\left(\omega_{\kappa}\left(\beta_{\kappa}(t)\right)-\omega_{\kappa}\left(\beta_{\kappa}^{*}\right)\right)\right|\\
&\le\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}|r_{\iota}(t)||\phi_{\iota\kappa}|\left|\omega_{\kappa}\left(\beta_{\kappa}(t)\right)-\omega_{\kappa}\left(\beta_{\kappa}^{*}\right)\right|\\
&\le\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\lambda_{\kappa}|\phi_{\iota\kappa}||r_{\iota}(t)||\hbar_{\kappa}(t)|\\
&\le\frac{1}{2}\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\lambda_{\kappa}|\phi_{\iota\kappa}|\left(r_{\iota}^{2}(t)+\hbar_{\kappa}^{2}(t)\right),
\end{aligned}
\tag{10}
$$

同理依次可得

$$
\begin{aligned}
&\sum_{\iota=1}^{n}|r_{\iota}(t)|\left|\bigwedge_{\kappa=1}^{m}\psi_{\iota\kappa}\left(\omega_{\kappa}\left(\beta_{\kappa}(t)\right)-\omega_{\kappa}\left(\beta_{\kappa}^{*}\right)\right)\right|\\
&\le\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}|r_{\iota}(t)||\psi_{\iota\kappa}|\left|\omega_{\kappa}\left(\beta_{\kappa}(t)\right)-\omega_{\kappa}\left(\beta_{\kappa}^{*}\right)\right|\\
&\le\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\lambda_{\kappa}|\psi_{\iota\kappa}||r_{\iota}(t)||\hbar_{\kappa}(t)|\\
&\le\frac{1}{2}\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\lambda_{\kappa}|\psi_{\iota\kappa}|\left(r_{\iota}^{2}(t)+\hbar_{\kappa}^{2}(t)\right),
\end{aligned}
\tag{11}
$$

$$\begin{aligned}&\sum_{\kappa=1}^{m}\left|\hbar_{\kappa}(t)\right|\left|\bigvee_{\iota=1}^{n}\delta_{\kappa\iota}\left(\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right)\right|\\&\le\sum_{\kappa=1}^{m}\sum_{\iota=1}^{n}\left|\hbar_{\kappa}(t)\right|\left|\delta_{\kappa\iota}\right|\left|\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right|\\&\le\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\chi_{\iota}\left|\delta_{\kappa\iota}\right|\left|r_{\iota}(t)\right|\left|\hbar_{\kappa}(t)\right|\\&\le\frac{1}{2}\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\chi_{\iota}\left|\delta_{\kappa\iota}\right|\left(r_{\iota}^{2}(t)+\hbar_{\kappa}^{2}(t)\right),\end{aligned}\tag{12}$$

$$\begin{aligned}&\sum_{\kappa=1}^{m}\left|\hbar_{\kappa}(t)\right|\left|\bigwedge_{\iota=1}^{n}\eta_{\kappa\iota}\left(\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right)\right|\\&\le\sum_{\kappa=1}^{m}\sum_{\iota=1}^{n}\left|\hbar_{\kappa}(t)\right|\left|\eta_{\kappa\iota}\right|\left|\varpi_{\iota}\left(\alpha_{\iota}(t)\right)-\varpi_{\iota}\left(\alpha_{\iota}^{*}\right)\right|\\&\le\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\chi_{\iota}\left|\eta_{\kappa\iota}\right|\left|r_{\iota}(t)\right|\left|\hbar_{\kappa}(t)\right|\\&\le\frac{1}{2}\sum_{\iota=1}^{n}\sum_{\kappa=1}^{m}\chi_{\iota}\left|\eta_{\kappa\iota}\right|\left(r_{\iota}^{2}(t)+\hbar_{\kappa}^{2}(t)\right).\end{aligned}\tag{13}$$

将(8)~(13)代入(7)，我们有

$$\begin{aligned}{}_{t_0}^{c}D_t^{\nu}V(t)\le&-\frac{1}{2}\sum_{\iota=1}^{n}\left[2\theta_{\iota}+2\ell_{\iota}+\chi_{\iota}\left(\left|\zeta_{\kappa\iota}\right|+\left|\delta_{\kappa\iota}\right|+\left|\eta_{\kappa\iota}\right|\right)-\sum_{\kappa=1}^{m}\lambda_{\kappa}\left(\left|\mu_{\iota\kappa}\right|+\left|\phi_{\iota\kappa}\right|+\left|\psi_{\iota\kappa}\right|\right)\right]r_{\iota}^{2}(t)\\&-\frac{1}{2}\sum_{\kappa=1}^{m}\left[2\vartheta_{\kappa}+2\wp_{\kappa}+\chi_{\iota}\left(\left|\zeta_{\kappa\iota}\right|+\left|\delta_{\kappa\iota}\right|+\left|\eta_{\kappa\iota}\right|\right)-\lambda_{\kappa}\sum_{\iota=1}^{n}\left(\left|\mu_{\iota\kappa}\right|+\left|\phi_{\iota\kappa}\right|+\left|\psi_{\iota\kappa}\right|\right)\right]\hbar_{\kappa}^{2}(t),\end{aligned}\tag{14}$$

令 $\Omega_1=\min\limits_{1\le\iota\le n}\left\{2\theta_{\iota}+2\ell_{\iota}+\chi_{\iota}\left(\left|\zeta_{\kappa\iota}\right|+\left|\delta_{\kappa\iota}\right|+\left|\eta_{\kappa\iota}\right|\right)-\sum_{\kappa=1}^{m}\lambda_{\kappa}\left(\left|\mu_{\iota\kappa}\right|+\left|\phi_{\iota\kappa}\right|+\left|\psi_{\iota\kappa}\right|\right)\right\}$，

$\Omega_2=\min\limits_{1\le\iota\le n}\left\{2\vartheta_{\kappa}+2\wp_{\kappa}+\chi_{\iota}\left(\left|\zeta_{\kappa\iota}\right|+\left|\delta_{\kappa\iota}\right|+\left|\eta_{\kappa\iota}\right|\right)-\lambda_{\kappa}\sum_{\iota=1}^{n}\left(\left|\mu_{\iota\kappa}\right|+\left|\phi_{\iota\kappa}\right|+\left|\psi_{\iota\kappa}\right|\right)\right\}$，$\Omega=\min\{\Omega_1,\Omega_2\}$，则由(14)可得

$${}_{t_0}^{c}D_t^{\nu}V(t)\le-\Omega\times\frac{1}{2}\left(\sum_{\iota=1}^{n}r_{\iota}^{2}(t)+\sum_{\kappa=1}^{m}\hbar_{\kappa}^{2}(t)\right)=-\Omega V(t).\tag{15}$$

对(15)使用引理 4 有

$$V(t)\le V(t_0)E_{\nu}\left(-\Omega(t-t_0)^{\nu}\right),\tag{16}$$

即

$$\left\|\xi-\xi^{*}\right\|^{2}\le\left\|\xi(t_0)-\xi^{*}\right\|^{2}E_{\nu}\left(-\Omega(t-t_0)^{\nu}\right),$$

其中 $\xi=(\alpha_1,\alpha_2,\cdots,\alpha_n,\beta_1,\beta_2,\cdots,\beta_m)^{\mathrm{T}}$，$\xi^{*}=\left(\alpha_1^{*},\alpha_2^{*},\cdots,\alpha_n^{*},\beta_1^{*},\beta_2^{*},\cdots,\beta_m^{*}\right)^{\mathrm{T}}$，由定义 3 可知分数阶 BAM 模糊神经网络(1)在平衡点处是全局 M-L 镇定的。

注 2. 当 $\nu=1$ 时，分数阶 BAM 模糊神经网络将退化为整数阶 BAM 模糊神经网络模型，此时定理 1 和 2 的结论仍成立。

注 3. 由于全局 M-L 镇定意味着全局渐近镇定，因此分数阶 BAM 模糊神经网络(1)在平衡点处也是全局渐近镇定的。

5. 总结与展望

本文研究了分数阶 BAM 模糊神经网络的全局 M-L 镇定，首先通过构造新的压缩映射并结合不等式

技巧与 2-范数分析方法严格证明了该模型平衡点的存在唯一性。此外，设计了一种简洁有效的线性反馈控制器，基于分数阶理论得到了分数阶 BAM 模糊神经网络实现全局 M-L 镇定的充分性准则。考虑到放大器有限的切换速度以及现实中不可避免的外部扰动，分析具有时滞与外部扰动的神经网络的动力学行为具有重要的应用前景，如何分析具有上述因素的分数阶 BAM 神经网络的动力学有待未来进一步探究。

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