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(2 + 1)维Hirota-Satsuma-Ito方程的怪波解及其动力学特征

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摘　要

本文基于Kadomtsev-Petviashvili约化方法，推导出(2 + 1)维Hirota-Satsuma-Ito方程的一般高阶怪波解。这些高阶怪波解以Gram行列式形式呈现。通过对怪波解的动力学分析，发现该方程的解具有暗－亮波结构，并揭示了相关参数对波形的叠加与分离模式的调控机制。

关键词

怪波，(2 + 1)维Hirota-Satsuma-Ito方程，KP约化技巧

Rogue Waves and Dynamics of Rogue Waves for the (2 + 1)-Dimensional Hirota-Satsuma-Ito Equation

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Abstract

In this work, we derive general high-order rogue waves of the (2 + 1)-dimensional Hirota-Satsuma-Ito equation by Kadomtsev-Petviashvili hierarchy reduction technique. These general high order rogue waves are expressed in terms of Gram determinants. Through dynamical analysis of the rogue wave solutions, it is found that the rogue waves of this equation exhibit dark-bright wave structures. Moreover, the regulatory mechanism of relevant parameters on the superposition and separation patterns of the waveforms is revealed.

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Keywords

Rogue Wave, (2 + 1)-Dimensional Hirota-Satsuma-Ito Equation, Kadomtsev-Petviashvili Hierarchy Reduction Technique

1. 引言

怪波最初在海洋学中被发现，随后被证实在非线性光学、玻色－爱因斯坦凝聚、超流体乃至金融等领域具有普适性[1]-[7]。从数学角度来看，怪波通常表现为可积系统的有理函数解，这类解在时间和空间上均具有局域特性。特别地，聚焦非线性薛定谔方程的一阶有理函数解——Peregrine 孤子，被公认为是描述怪波现象的经典模型[8] [9]。进一步研究表明，该方程的高阶有理解能够有效刻画振幅更大的怪波现象[10]。为此，研究者们发展了多种方法来构造可积系统的高阶有理解[11]-[19]。

在众多方法中，双线性方法与 Kadomtsev-Petviashvili 约化技巧的结合被证明是获取双线性可积系统高阶怪波解的有效工具。该技术已成功应用于非线性薛定谔方程[20]、Yajima-Oikawa 系统[21]、三波共振相互作用系统[22]、Sasa-Satsuma 方程[23]、矢量非线性薛定谔方程[24]等多个模型[25]-[28]。近期，叶梁荣等人改进了 Kadomtsev-Petviashvili 约化方法，并将其应用于(3 + 1)维 Yu-Toda-Sasa-Fukuyana 方程高阶怪波解的构造[29]。

本文聚焦于(2 + 1)维 Hirota-Satsuma-Ito (HSI)方程

$$u_{xx}+u_{ty}+\left(3u_tu_x\right)_x+u_{txxx}=0, \tag{1}$$

该方程是流体力学中一个重要的非线性系统，用于描述单向浅水波传播以及具有不同色散关系的长波相互作用[30] [31]。目前，方程(1)已有多种解被报道，包括复子解[32]、孤子解[33]-[36]、呼吸子解[37]、相互作用解[38]-[41]以及一些低阶有理函数解[42]-[45]。但是，文献[42]-[45]所给出的有理解主要集中在低阶解的情形，他们运用不同方法从理论上得到一阶和二阶有理解，由于高阶计算更加复杂，尚未形成适用于任意阶数的形式。与之相比，本文所构造的怪波解在阶数上实现了从低阶到 N 阶的推广。另外，方程(1)的一般高阶有理函数解可以通过长波极限法构造[37]，即在每一阶解获得过程中都需要通过取极限确定特定的参数取值，实际操作较为繁琐，且难以给出统一的表达式。相比之下，本文通过双线性方法与 KP 约化技巧的结合，成功构造了(2 + 1)维 HSI 方程(1)的高阶显式怪波解，且其以 Gram 行列式的简洁形式呈现。

本文的结构安排如下：在第二节给出了(2 + 1)维 HSI 方程(1)的双线性形式，并应用 KP 约化方法推导出一般的 N 阶怪波解。第三节详细阐述怪波解的推导过程。第四节重点分析怪波解的动力学特性。最后，第五节总结全文。

2. 一般怪波解

我们首先引入变量变换

$$u=2\left(\ln f\right)_x, \tag{2}$$

将式子(2)代入(1)可得到双线性形式

$$\left(D_x^3D_t+D_yD_t+D_x^2\right)f\cdot f=0, \tag{3}$$

其中符号 D 表示 Hirota 双线性微分算子[17]，其定义为

$$D_x^{m_1}D_y^{m_2}D_t^{m_3}F(x,y,t)\cdot G(x,y,t)=\left(\partial_x-\partial_{x'}\right)^{m_1}\left(\partial_y-\partial_{y'}\right)^{m_2}\left(\partial_t-\partial_{t'}\right)^{m_3}\times F(x,y,t)\cdot G(x',y',t')\big|_{x'=x,y'=y,t'=t}.$$

借助双线性形式(3)和 KP 约化技巧，我们推导出(2 + 1)维 HSI 方程(1)的一般怪波解。为了给出这些解的显式表达式，我们引入 Schur 多项式如下

$$\sum_{r=0}^{\infty}S_r(x)\kappa^r=\exp\left(\sum_{r=1}^{\infty}x_r\kappa^r\right),$$

这里 $\boldsymbol{x}=(x_1,x_2,\cdots)$。它可以具体表示为

$$S_0(x)=1,\ S_1(x)=x_1,\ S_2(x)=\frac{1}{2}x_1^2+x_2,\cdots,\ S_r(x)=\sum_{r_1+2r_2+\cdots+nr_n=r}\left(\prod_{i=1}^{n}\frac{x_i^{r_i}}{r_i!}\right).$$

定理：(2 + 1)维 HSI 方程(1)存在一般形式的高阶怪波解

$$u=2(\ln f)_x, \tag{4}$$

这里

$$f=\det_{1\le i,j\le N}\left(m_{2i-1,2j-1}\right),$$
$$m_{i,j}=\sum_{v=0}^{\min(i,j)}\left(\frac{1}{12}\right)^v S_{i-v}\left(\boldsymbol{x}^++v\boldsymbol{s}\right)S_{j-v}\left(\boldsymbol{x}^-+v\boldsymbol{s}^*\right). \tag{5}$$

下文中的符号“*”表示复共轭。向量 $\boldsymbol{x}^\pm=\left(x_1^\pm,x_2^\pm,\cdots\right)$ 的分量定义为

$$x_{2r}^+=0,\ x_{2r+1}^+=\alpha_{2r+1}x+\beta_{2r+1}t+\gamma_{2r+1}y+a_{2r+1},$$
$$x_{2r}^-=0,\ x_{2r+1}^-=\alpha_{2r+1}^*x+\beta_{2r+1}^*t+\gamma_{2r+1}^*y+a_{2r+1}^*,$$

其中系数 $\alpha_r,\beta_r,\gamma_r$ 由以下展开式确定

$$p(\kappa)-p_0=\sum_{r=1}^{\infty}\alpha_r\kappa^r,$$

$$\frac{1}{12i}\left(p(\kappa)^2-p_0^2\right)=\sum_{r=1}^{\infty}\beta_r\kappa^r,$$

$$2\left(p(\kappa)^3-p_0^3\right)=\sum_{r=1}^{\infty}\gamma_r\kappa^r,$$

此外，还满足

$$p^4(\kappa)-4ip(\kappa)=G(p_0)\cosh(\kappa),\ G(p)=p^4-4ip,\ p_0=\frac{\sqrt{3}}{2}+\frac{i}{2}.$$

向量 $\boldsymbol{s}=(s_1,s_2,\cdots)$ 的分量由以下表达式确定

$$\ln\left[\frac{1}{\kappa}\left(\frac{\left(p_0+p_0^*\right)\left(p(\kappa)-p_0\right)}{\left(p(\kappa)+p_0^*\right)p_1}\right)\right]=\sum_{r=1}^{\infty}s_r\kappa^r,\ p_1\equiv\frac{\mathrm{d}p}{\mathrm{d}\kappa}\bigg|_{\kappa=0},$$

其中 $a_{2r+1}\ (r=0,1,2,\cdots)$ 为任意常数。

注记：上述定理中函数 $p(\kappa)$ 的展开式构造方法与研究[22]类似。

3. 怪波解的推导

本节将借助双线性方程(3)和 KP 约化方法，推导(2 + 1)维 HSI 方程(1)的怪波解。首先给出以下引理。

引理：KP 族中的双线性方程

$$\left(D_{x_1}^3 D_{x_2} + 2D_{x_2}D_{x_3} - 3D_{x_1}D_{x_4}\right)\tau\cdot\tau = 0, \tag{6}$$

存在有理解

$$\tau = \det_{1\le\mu,\nu\le N}\left(m_{i_\mu,j_\nu}\right),$$

其中 N 为非负整数，$\left(i_1,i_2,\cdots,i_N;j_1,j_2,\cdots,j_N\right)$ 为任意的指标序列。矩阵元 $m_{i,j}^{(n)}$ 定义为

$$\begin{aligned}
m_{i,j} &= \frac{\left[f_1(p)\partial_p\right]^i}{i!}\frac{\left[f_2(q)\partial_q\right]^j}{j!}\frac{1}{p+q}\mathrm{e}^{\xi+\eta},\\
\xi &= px_1 + p^2x_2 + p^3x_3 + p^4x_4 + \xi_0(p),\\
\eta &= qx_1 - q^2x_2 + q^3x_3 - q^4x_4 + \eta_0(q),
\end{aligned}$$

其中 $f_1(p)$、$\xi_0(p)$ 与 $f_2(q)$、$\eta_0(q)$ 分别为变量 p 和 q 的函数。

若约化条件及其复共轭关系

$$\left(\partial_{x_4} - ai\partial_{x_1}\right)\tau = C\tau, \tau = \tau^*, \tag{7}$$

被满足。接着取 $\tau = f$，双线性方程(6)可化为

$$\left(D_{x_1}^3 D_{x_2} + 2D_{x_2}D_{x_3} - 3aiD_{x_1}^2\right)f\cdot f = 0, \tag{8}$$

其中 a 为任意实常数，C 为任意复常数。进一步，通过坐标变换

$$x_1 = x, x_2 = -\frac{t}{3ai}, x_3 = 2y,$$

此时(8)化为，

$$\left(D_x^3 D_t + D_y D_t + D_x^2\right)f\cdot f = 0.$$

此即(2 + 1)维 HSI 方程(1)的双线性形式(3)。

根据文献[29]，约化条件(7)可导出

$$\tau = \det_{1\le i,j\le N}\left(m_{2i-1,2j-1}\big|_{p=p_0,q=p_0^*}\right), \tag{9}$$

其中矩阵元素

$$\begin{aligned}
m_{i,j} &= \frac{\left[f_1(p)\partial_p\right]^i}{i!}\frac{\left[f_2(q)\partial_q\right]^j}{j!}\frac{1}{p+q}\mathrm{e}^{\xi+\eta},\\
\xi &= px + 2p^3y - \frac{1}{3ai}p^2t + \sum_{r=1}^{\infty}a_r\ln^r w_1(p),\\
\eta &= qx + 2q^3y + \frac{1}{3ai}q^2t + \sum_{r=1}^{\infty}a_r^*\ln^r w_2(q).
\end{aligned} \tag{10}$$

另外，

$$G_1(p) = p^4 - iap, G_2(q) = -q^4 - iaq,$$

$$p_0=\frac{a^{\frac{1}{3}}}{2^{\frac{5}{3}}}\left(\sqrt{3}+i\right),\ f_1(p)=\pm\frac{\sqrt{G_1^2(p)-G_1^2(p_0)}}{G_1'(p)},$$

$$q_0=\frac{a^{\frac{1}{3}}}{2^{\frac{5}{3}}}\left(\sqrt{3}-i\right),\ f_2(q)=\pm\frac{\sqrt{G_2^2(q)-G_2^2(q_0)}}{G_2'(q)}.$$

怪波解的具体形式依赖于 $G'(p)=0$ 的根结构，其中 $G'(p)$ 表示 $G(p)$ 关于 p 的导数。在此情形下，方程 $G'(p)=0$ 存在三个不同的根：

$$p_{01}=\frac{a^{\frac{1}{3}}}{2^{\frac{5}{3}}}\left(\sqrt{3}+i\right),\ p_{02}=\frac{a^{\frac{1}{3}}}{2^{\frac{5}{3}}}\left(\sqrt{3}-i\right),\ p_{03}=-\frac{a^{\frac{1}{3}}}{2^{\frac{2}{3}}}i.$$

显然，纯虚根 p_{03} 将导致解(9)中的元素 $m_{i,j}$ 出现奇异性。同时，p_{01} 与 p_{02} 由于共轭会产生相同的表达式。因此，本文选取 $p_0=p_{01}$。另外，为简便起见，取常数参数 $a=4$。

在此情形下，根据文献[27]，当

$$\xi_0=\sum_{r=1}^{\infty}a_r\ln^r w_1(p)=\eta_0^*=\sum_{r=1}^{\infty}a_r^*\ln^r w_2(q),$$

$$w_1(p)=\frac{G_1(p)\pm\sqrt{G_1^2(p)-G_1^2(p_0)}}{G_1(p_0)},\ w_2(q)=\frac{G_2(q)\pm\sqrt{G_2^2(q)-G_2^2(p_0^*)}}{G_2(p_0^*)},$$

自然可得

$$\left(m_{i,j}\big|_{p=p_0,q=p_0^*}\right)^*=m_{j,i}\big|_{p=p_0,q=p_0^*},$$

这意味着 $\tau=\tau^*$ 成立。同时，τ 的矩阵元由(10)给出。最后，借助文献[20]中的简化方法，定理的证明得以完成。

奇异性分析：根据 Melnikov 方程正则性证明的方法[28]，可推导出(2 + 1)维 HSI 方程(1)的 N 阶有理解是非奇异的。由引理可知，$f=\tau$ 能写成一个 Hermite 矩阵

$$\boldsymbol{M}=\left(m_{2i-1,2j-1}\right)_{i,j=1}^{N},$$

$$m_{i,j}=\frac{\left[f_1(p)\partial_p\right]^i}{i!}\frac{\left[f_2(q)\partial_q\right]^j}{j!}\int_{-\infty}^{x}e^{\xi+\eta}dx\Big|_{p=p_0,q=q_0}.$$

因此，对于任意非零向量 $\boldsymbol{v}=(v_1,v_2,\cdots,v_N)$ 及其共轭转置 $\overline{\boldsymbol{v}}$，有

$$\begin{aligned}\boldsymbol{vM\overline{v}}&=\sum_{i,j=1}^{N}v_i m_{i,j}\overline{v}_j\\&=\sum_{i,j=1}^{N}v_i\frac{\left[f_1(p)\partial_p\right]^i}{i!}\frac{\left[f_2(q)\partial_q\right]^j}{j!}\int_{-\infty}^{x}e^{\xi+\eta}dx\Big|_{p=p_0,q=q_0}\overline{v}_j\\&=\int_{-\infty}^{x}\sum_{i,j=1}^{N}v_i\frac{\left[f_1(p)\partial_p\right]^i}{i!}\frac{\left[f_2(q)\partial_q\right]^j}{j!}e^{\xi+\xi^*}\Big|_{p=p_0,q=q_0}\overline{v}_j dx\\&=\int_{-\infty}^{x}\left|\sum_{i=1}^{N}v_i\frac{\left[f_1(p)\partial_p\right]^i}{i!}e^{\xi}\Big|_{p=p_0}\right|^2dx.\end{aligned}$$

这意味着 Hermitian 矩阵是正定的，即 f 对所有的 (x,y,t) 都是非零的。因此，(2 + 1)维 HSI 方程(1)的 N 阶有理解是非奇异的。

4. 怪波的动力学

本节将系统讨论(2 + 1)维 HSI 方程(1)怪波解的动力学行为。

4.1. 一阶怪波解

为得到一阶怪波解，在定理中取 $N=1$，$a_1=0$，则(2 + 1)维 HSI 方程(1)的一阶怪波解为

$$u=2\left(\ln f_1\right)_x, \tag{11}$$

其中

$$f_1=\frac{1}{4}x^2+9y^2+\frac{1}{144}t^2+\frac{3}{2}xy-\frac{1}{24}xt+\frac{1}{4}yt+\frac{1}{12}.$$

当 $t=0$ 时，解(11)在 (x,y) 平面上呈现如图 1 所示的空间局部化 lump 解。本文主要关注怪波解的构造与动力学特性，接下来将进一步讨论时间局部化的怪波解。

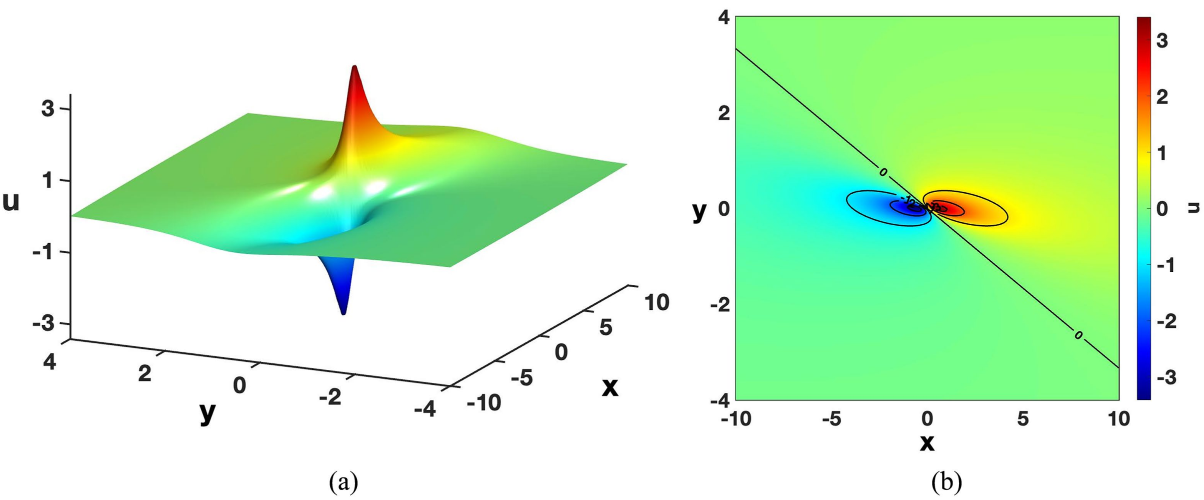


Figure 1. First-order lump of (1) at $t=0$: (a) Three-dimensional plot; (b) Contour plot
图 1. 方程(1)在 $t=0$ 时的一阶怪波解：(a) 三维立体图；(b) 平面图

一阶怪波解(11)在 (x,t) 平面上存在如下临界点

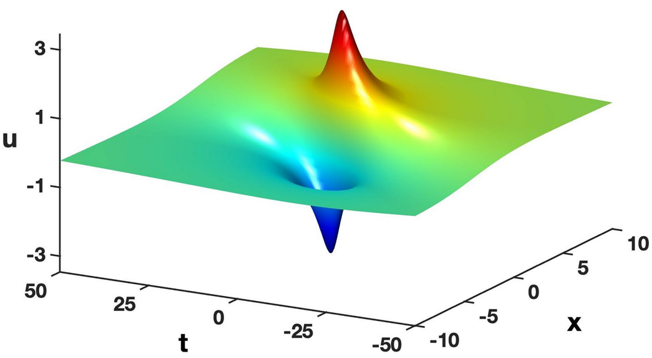
$$\left(x_1,t_1\right)=\left(-4y+\frac{\sqrt{3}}{6}\sqrt{144y^2+4},-12y\right),$$

$$\left(x_2,t_2\right)=\left(-4y-\frac{\sqrt{3}}{6}\sqrt{144y^2+4},-12y\right),$$

根据局域分析，一阶怪波解(11)呈现暗–亮怪波模式。具体而言，u 在点 $\left(x_1,t_1\right)$ 处取得局部极大值，在点 $\left(x_2,t_2\right)$ 处取得局部极小值，体现了(2 + 1)维 HSI 方程的波型模式。进一步分析发现，该解在空间上具有局域化特征：在极大值点附近呈现亮波隆起，而在极小值点附近则形成暗波凹陷，二者构成完整的暗–亮波结构。另外，对于不同的 y 值，在 (x,t) 平面上，他的上述临界值虽会发生平移，但临界值之间的相对位置保持不变。因此，y 值对一阶怪波解(6)的波形并无本质影响，仅引起微小的相位变化，其振

幅和波形轮廓保持稳定。为简便起见，本文取 $y=0$ 进行讨论。

图 2 展示了(2 + 1)维 HSI 方程(1)的一阶怪波解图像，可以看到如局域分析所述，局部极大值处有一个显著的隆起，对侧分布着一个凹陷区域，这种结构正是暗－亮怪波的典型特征。等高线图(b)能更加直观地看出波形的局域化特性，能量主要集中在极值并向四周迅速衰减。



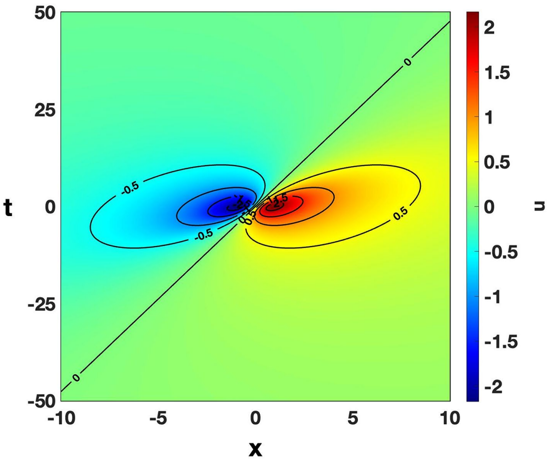


Figure 2. First-order rogue waves of (1) at $y=0$: (a) Three-dimensional plot; (b) Contour plot

图 2. 方程(1)在 $y=0$ 时的一阶怪波解：(a) 三维立体图；(b) 平面图

4.2. 二阶怪波解

为得到二阶怪波解，在定理中取 $N=2$，$a_1=a_{11}+a_{12}i$，$a_3=a_{31}+a_{32}i$，则解的形式为

$$u=2\left(\ln f_2\right)_x, \tag{12}$$

其中

$$m_{1,1}=x_1^+x_1^-+\lambda,$$
$$m_{1,3}=x_1^+\left[\frac{1}{6}\left(x_1^-\right)^3+x_3^-\right]+\lambda\left[\frac{1}{2}\left(x_1^-+s_1^*\right)^2+s_2^*\right],$$
$$m_{3,1}=\left[\frac{1}{6}\left(x_1^+\right)^3+x_3^+\right]x_1^-+\lambda\left[\frac{1}{2}\left(x_1^++s_1\right)^2+s_2\right],$$
$$m_{3,3}=\left[\frac{1}{6}\left(x_1^+\right)^3+x_3^+\right]\left[\frac{1}{6}\left(x_1^-\right)^3+x_3^-\right]+\lambda\left[\frac{1}{2}\left(x_1^++s_1\right)^2+s_2\right]\left[\frac{1}{2}\left(x_1^-+s_1^*\right)^2+s_2^*\right]$$
$$+\lambda^2\left(x_1^-+2s_1\right)\left(x_1^++2s_1^*\right)+\lambda^3.$$

这里，

$$x_{2r+1}^+=\alpha_{2r+1}x+\beta_{2r+1}t+\gamma_{2r+1}y+a_{2r+1},$$
$$x_{2r+1}^-=\alpha_{2r+1}^*x+\beta_{2r+1}^*t+\gamma_{2r+1}^*y+a_{2r+1}^*,$$

有 $r=0,1$ 并且

$$\alpha_1=-\frac{1}{4}\left(\sqrt{3}-i\right),\alpha_3=\frac{1}{576}\left(\sqrt{3}i-1\right),$$

$$\beta_1=\frac{1}{2}\left(\sqrt{3}-i\right),\beta_3=\frac{11}{288}\left(\sqrt{3}-i\right),$$

$$\gamma_1=-\frac{3}{2},\gamma_3=\frac{11}{96},$$

$$s_1=-\frac{1}{12}\left(\sqrt{3}-5i\right),s_2=-\frac{1}{18}-\frac{5\sqrt{3}}{144}i,\lambda=\frac{1}{12}.$$

为实现二阶怪波解的最优叠加，对解施加对称性条件 $f_2(x,t)=f_2(-x,-t)$ 并在 $y=0$ 处进行验证。通过令等式两端的系数对应相等，得到参数 a_1 和 a_3 满足的代数约束方程组。求解该方程组可以唯一确定其取值为

$$a_1=-\frac{\sqrt{3}}{12}+\frac{5}{12}i,a_3=\frac{7\sqrt{3}}{576}-\frac{35}{5184}i.$$

基于对一阶怪波解的分析，我们进一步研究二阶怪波解的动力学行为。二阶怪波解(12)的最优叠加图像如图 3 所示。从图中可以看出，二阶怪波的振幅高于一阶怪波，这体现了高阶局域激发所带来的能量增强效应。其次，在最优叠加情形下，即满足对称性条件时，多个暗－亮怪波结构相互叠加，形成复杂的波包络。此时各子波紧密排列，使得每个暗－亮怪波的完整轮廓难以清晰分辨。这种叠加态体现了系统高阶激发的复杂性。

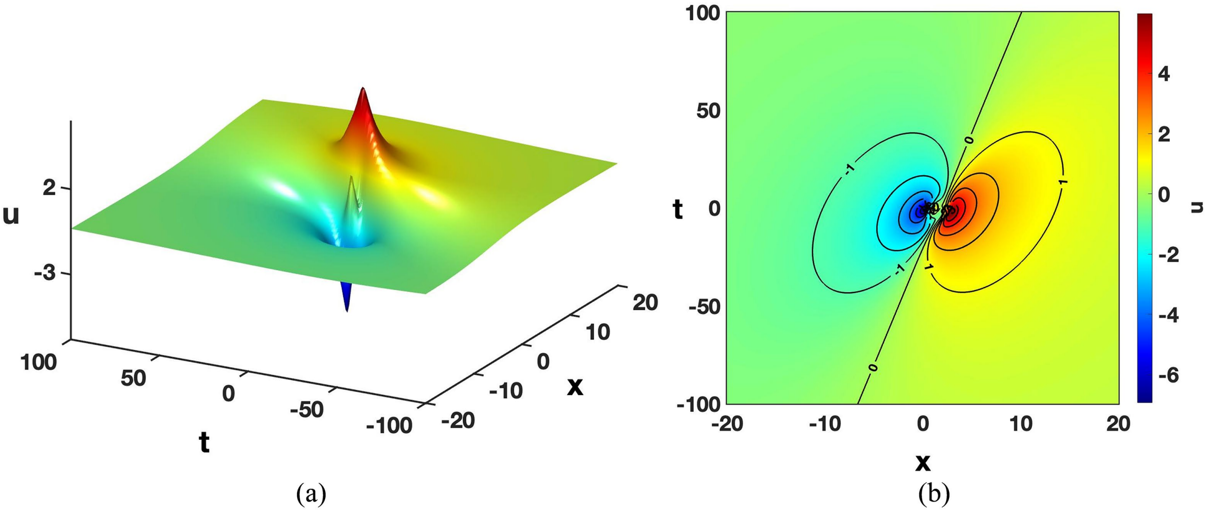
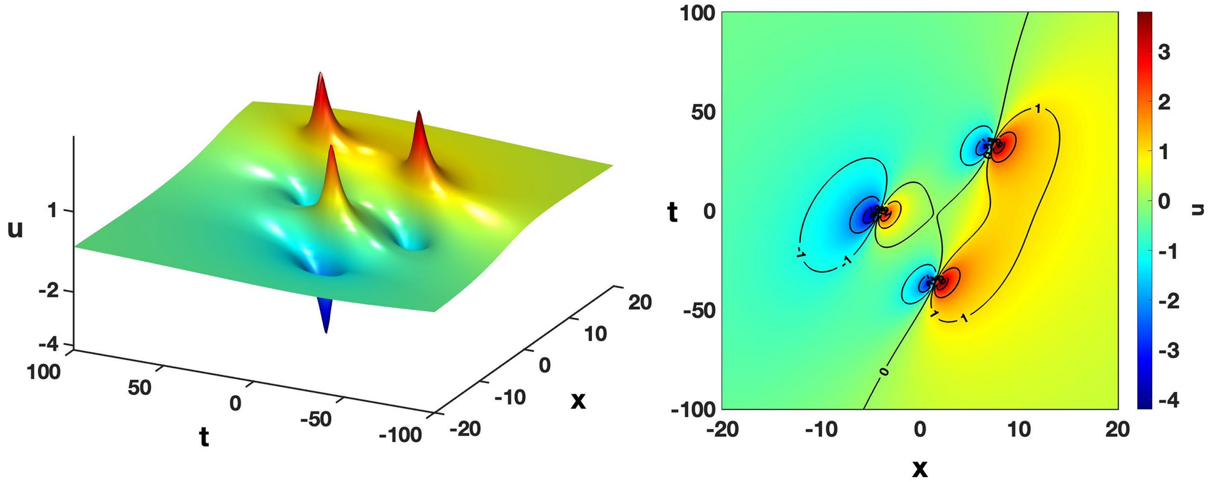


Figure 3. Second-order optimal superposition rogue waves of (1) at $y=0$: (a) Three-dimensional plot; (b) Contour plot
图 3. 方程(1)在 $y=0$ 时的二阶最优叠加怪波解：(a) 三维立体图；(b) 平面图

为揭示各子波的结构，可在参数 a_3 中加入一个非零常数，即打破原有的对称性条件。如图 4 所示，当取 $a_3=\frac{7\sqrt{3}}{576}-\frac{35}{5184}i+8$ 时，原本紧密叠加的暗－亮怪波发生分离，形成三个清晰可辨的暗－亮波对。这些子波在空间上呈三角形分布，每个子波均保持与一阶解相似的暗－亮结构。这一现象表明，参数 a_3 在调控二阶怪波解中起着关键作用，能够有效控制波的分离程度与叠加模式：当 a_3 为最优叠加情况时，各子波完全叠加；随着所加这一非零常数绝对值的增大，子波逐渐分离；当这一常数足够大时，子波完全分离并各自独立演化。

类似地，我们可以进一步推导出(2 + 1)维 HSI 方程(1)的三阶及更高阶怪波解。这些高阶解在形式上

更为复杂，其动力学行为也更为丰富。高阶怪波解可视为基本怪波模在不同参数下的非线性叠加，通过调节参数，可以实现从完全叠加到完全分离的连续过渡。



(a) (b)

Figure 4. Second-order rogue waves of (1) at $y=0$ and $a_3=\frac{7\sqrt{3}}{576}-\frac{35}{5184}i+8$: (a) Three-dimensional plot; (b) Contour plot

图 4. 方程(1)在 $y=0$ 和 $a_3=\frac{7\sqrt{3}}{576}-\frac{35}{5184}i+8$ 时的二阶怪波解：(a) 三维立体图；(b) 平面图

5. 总结

本文基于双线性方法和 KP 约化技巧，系统构造了(2 + 1)维 Hirota-Satsuma-Ito 方程的高阶怪波解。所得高阶怪波解以 Gram 行列式形式给出，形式简洁、结构清晰，为进一步研究该方程的非线性波现象提供了理论基础。另外，通过对一阶和二阶怪波解的动力学分析，揭示了该方程怪波解的特征，即解具有暗－亮波结构，另外，给出了暗－亮模式的叠加与分离规律，当参数满足对称性条件时，各子波完全叠加，当参数加上一个非零常数时，叠加的子波能分离开，使得暗－亮怪波各自的轮廓变得清晰可辨。

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